Roller diameter = 44 mm
Angle of ascent = 60°

Calculate the acceleration of a follower at the beginning of the lift. Also, find its values when the roller just touches the nose and is at the apex of the circular nose. Sketch the variation of displacement, velocity and acceleration during ascent.

(88.12 m/s²; 164 m/s²; and −92.6 m/s²; −111 m/s²)

23. A flat-ended valve tappet is operated by a symmetrical cam with circular arcs for flank and nose profiles. The total angle of action is 150°, base circle diameter is 125 mm and the lift is 25 mm. During the lift, the period of acceleration is half that of the declaration. The speed of cam shift is 1250 rpm. The straight-line path of the tappet passes through the cam axis. Find
    (i) radii of the nose and the flank, and
    (ii) maximum acceleration and declaration during the lift.

(40.3 mm, 148 mm; 1465 m/s²; 808.8 m/s²)

24. In a four-stroke petrol engine, the crank angle is 5° after t.d.c. when the suction valve opens and 53° after b.d.c. when the suction valve closes. The lift is 8 mm, the nose radius is 3 mm and the least radius of the cam is 18 mm. The shaft rotates at 800 rpm. The cam is of the circular type with a circular nose and flanks while the follower is flat-faced. Determine the maximum velocity and the maximum acceleration and retardation of the valve. When is the minimum force exerted by the springs to overcome the inertia of moving parts weighting 250 g.

(1.3 m/s; 433.7 m/s; 161.4 m/s²; 40.35 N)

25. A symmetrical circular cam operates a flat-faced follower with a lift of 30 mm. The minimum radius of the cam is 50 mm and the nose radius is 12 mm. The angle of lift is 80°. If the speed of the cam is 210 rpm, find the main dimensions of the cam and the acceleration of the follower at (i) the beginning of the lift (ii) the end of contact with the circular flank (iii) the beginning of contact with the nose, and (iv) the apex of nose.

(r = 68 mm, 29.38 m/s², 23.8 m/s², 26.2 m/s², 32.9 m/s²)

26. A circular disc cam with diameter of 80 mm with its centre displaced at30 mm from the camshaft is used with a flat surface follower. The line of action of the follower is vertical and passes through the shaft axis. The mass of the follower is 2.5 kg and is pressed downwards with a spring of stiffness 4 N/mm. In the lowest position, the spring force is 50 N. Derive an expression for the acceleration of the follower as a function of cam rotation from the lowest position of the follower. Also, find the speed at which the follower begins to lift from the cam surface.

(618.4 rpm)

8 FRICTION

Introduction

When a body slides over another, the motion is resisted by a force called the *force of friction*. The force arises from the fact that the surfaces, though planed and made smooth, have ridges and depressions that interlock and the relative movement is resisted. Thus, the force of friction on a body is parallel to the sliding surfaces and acts in a direction opposite to that of the sliding body.

There are phenomena, where it is necessary to reduce the force of friction whereas in some cases it must be increased. In case of lathe slides, journal bearings, etc., where the power transmitted is reduced due to friction, it has to be decreased by the use of lubricated surfaces. In processes where the power is transmitted through friction, attempts are made to increase it to transmit more power. Examples are friction clutches and belt drives. Even the tightness of a nut and bolt is dependent mainly on the force of friction. Had there been no friction between the nut and the surface on which it is tightened, the nut would loosen off at the moment the spanner is removed after tightening.

8.1 KINDS OF FRICTION

Usually, three kinds of friction, depending upon the conditions of surfaces are considered.

1. Dry Friction

Dry friction is said to occur when there is relative motion between two completely unlubricated surfaces. It is further divided into two types:

(a) ***Solid Friction*** When the two surfaces have a sliding motion relative to each other, it is called a solid friction.

(b) ***Rolling Friction*** Friction due to rolling of one surface over another (e.g., ball and roller bearings) is called rolling friction (Sec 8.10).

2. Skin or Greasy Friction

When the two surfaces in contact have a minute thin layer of lubricant between them, it is known as *skin* or *greasy friction*. Higher spots on the surface break through the lubricant and come in contact with the other surface.

Skin friction is also termed as *boundary* friction (Sec 8.12).

3. Film Friction

When the two surfaces in contact are completely separated by a lubricant, friction will occur due to the shearing of different layers of the lubricant. This is known as *film friction* or *viscous* friction (Sec. 8.15).

8.2 LAWS OF FRICTION

Experiments have shown that the force of solid friction

- is directly proportional to normal reaction between the two surfaces
- opposes the motion between the surfaces
- depends upon the materials of the two surfaces
- is independent of the area of contact
- is independent of the velocity of sliding

The last of these laws is not true in the strict sense as it has been found that the friction force decreases slightly with the increase in velocity.

8.3 COEFFICIENT OF FRICTION

Let a body of weight **W** rest on a smooth and dry plane surface. Under the circumstances, the plane surface also exerts a reaction force $\mathbf{R}_n$ on the body which is normal to the plane surface. If the plane surface considered is horizontal, $\mathbf{R}_n$ would be equal and opposite to **W** [Fig. 8.1(a)].

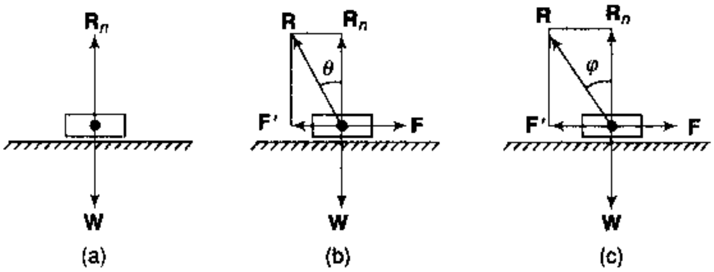


Fig. 8.1

Let a small horizontal force F be applied to the body to move it on the surface [Fig. 8.1(b)]. So long the body is unable to move, the equilibrium of the body provides

$$R_n = W \quad \text{and} \quad F = F'$$

where **F′** is the horizontal force resisting the motion of the body. As the force **F** is increased, the relative force **F′** also increases accordingly. **F′** and $\mathbf{R}_n$, the friction and the normal reaction forces can also be combined into a single reaction force **R** inclined at an angle θ to the normal. Thus

$$R\cos\theta = W \quad \text{and} \quad R\sin\theta = F$$

At a moment, when the force **F** would just move the body, the value of **F′** or **R** $\sin\theta$ (equal to **F**) is called the *static force of friction*. Angle θ attains the value φ and the body is in equilibrium under the action of three forces [Fig. 8.1(c)]:

F, in the horizontal direction

W, in the vertical downward direction, and

R, at an angle φ with the normal (inclined towards the force of friction).

$$F' \propto R_n$$
$$= \mu R_n$$

where μ is known as the *coefficient of friction*.

or

$$\mu = \frac{F'}{R_n}$$

Also, in Fig. 8.1(c), $\tan\varphi = \dfrac{F'}{R_n}$

or

$$\tan\varphi = \frac{\mu R_n}{R_n} = \mu \qquad (8.1)$$

The angle φ is known as the *limiting angle of friction* or simply the *angle of friction*.

Now, if the body moves over the plane surface, it is observed that the friction force will be slightly less than the static friction force. As long as the body moves with a uniform velocity, the force F required for the motion of the body will be equal to the force of friction on the body. However, if the velocity is to increase, additional force will be needed to accelerate the body. Thus, while the body is in motion, it can be written that

$$\tan\varphi = \mu$$

where φ is approximately the limiting angle of friction.

Also, no movement is possible until the angle of reaction **R** with the normal becomes equal to the limiting angle of friction or until $\varphi = \mu$.

Example 8.1 ***The force required just to move a body on a rough horizontal surface by pulling is 320 N inclined at 30° and by pushing 380 N at the same angle. Find the weight of the body and the coefficient of friction.***

Solution

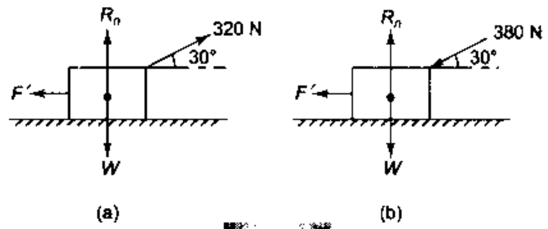


Fig. 8.2

(a) Consider the pull [Fig. 8.2(a)].
Resolving the forces horizontally,
$F' = 320 \cos 30°$

or $\mu R_n = 277$ or $R_n = 277/\mu$

Resolving the forces vertically,

$R_n + 320 \sin 30° = W$

or $\dfrac{277}{\mu} + 160 = W$ or $\mu = \dfrac{277}{W - 160}$ (i)

Similarly, consider the push [Fig. 8.2(b)],
Resolving the forces horizontally,
$F' = 380 \cos 30°$ or $\mu R_n = 329$ or $R_n = 329/\mu$
Resolving the forces vertically,

$R_n = W + 380 \sin 30°$

or $\dfrac{329}{\mu} = W + 190$ or $\mu = \dfrac{329}{W + 190}$ (ii)

Equating (i) and (ii),

$$\frac{277}{W - 160} = \frac{329}{W + 190}$$

or $277\,(W + 190) = 329\,(W - 160)$
or $52\,W = 105\,270$ or $W = 2024.4$ N
or $\mu = 0.1486$

8.4 INCLINED PLANE

1. Body at Rest

When a body is at rest on an inclined plane making an angle α with the horizontal, the forces acting on the body are (Fig. 8.3)

- **W**, weight of body in downward direction
- $\mathbf{R}_n$, normal reaction
- **F′**, force resisting the motion of body

From equilibrium conditions,

$$W \sin\alpha = F' \text{ and } W\cos\alpha = R_n$$

If the angle of inclination of the plane is increased, the body will just slide down the plane of its own when

$$W\sin\alpha = F' = \mu R_n = \mu W\cos\alpha$$

or $\quad \tan\alpha = \mu = \tan\varphi$

or $\quad \alpha = \varphi \qquad (8.2)$

This maximum value of the angle of inclination of the plane with the horizontal when the body starts sliding on its own is known as the *angle of repose* or *limiting angle of friction*.

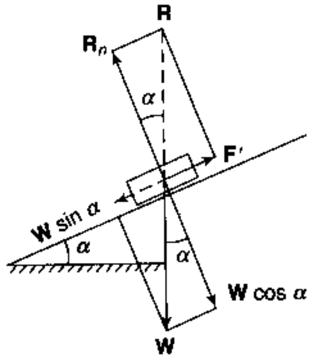


Fig. 8.3

2. Motion Up the Plane

Consider a body moving up an inclined plane under the action of a force **F** as shown in Fig. 8.4(a).

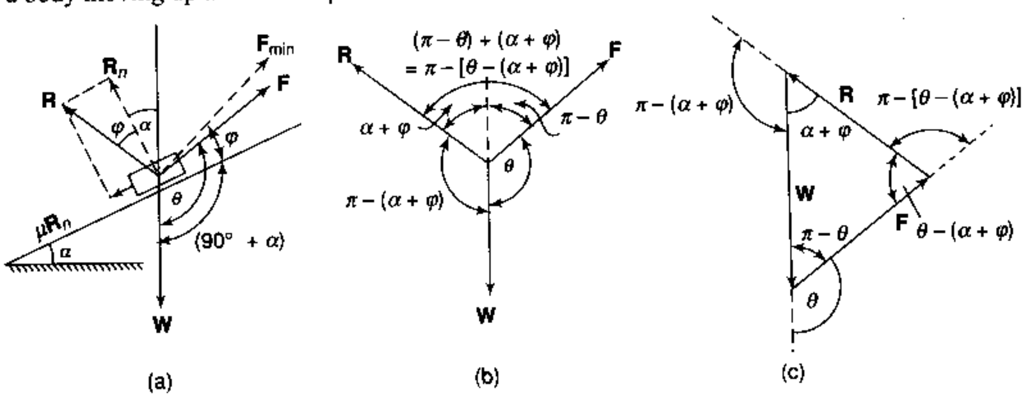


Fig. 8.4

Let α = angle of inclined plane with the horizontal

θ = angle of force **F** with the direction of **W**.

As the motion is up the plane, the friction force $\mathbf{F}'$ ($= \mu\mathbf{R}_n$) would act downwards along the plane. Combining $\mathbf{F}'$ and $\mathbf{R}_n$ as before, the body is in equilibrium under the action of forces **F**, **W** and **R** [Fig. 8.4(b)].

Applying Lami's theorem, we get

$$\frac{F}{\sin[\pi - (\alpha + \varphi)]} = \frac{W}{\sin[\pi - \{\theta - (\alpha + \varphi)\}]}$$

$$\frac{F}{\sin(\alpha + \varphi)} = \frac{W}{\sin[\theta - (\alpha + \varphi)]}$$

Alternatively, a triangle of forces can be drawn as shown in Fig. 8.4(c), and applying sine law of forces,

$$\frac{F}{\sin(\alpha+\varphi)}=\frac{W}{\sin[\theta-(\alpha+\varphi)]}$$

Thus,

$$F=\frac{W\sin(\alpha+\varphi)}{\sin[\theta-(\alpha+\varphi)]} \qquad (8.3)$$

(i) if the force applied is horizontal, $\theta = 90^\circ$

$$F=\frac{W\sin(\alpha+\varphi)}{\sin[90^\circ-(\alpha+\varphi)]}=\frac{W\sin(\alpha+\varphi)}{\cos(\alpha+\varphi)}=W\tan(\alpha+\varphi)$$

(ii) if the force applied is parallel to the plane, $\theta = 90^\circ + \alpha$

$$\begin{aligned}F&=\frac{W\sin(\alpha+\varphi)}{\sin\left[90^\circ+\alpha-(\alpha+\varphi)\right]}\\&=\frac{W\sin(\alpha+\varphi)}{\cos\varphi}\\&=\frac{W(\sin\alpha\cos\varphi+\cos\alpha\sin\varphi)}{\cos\varphi}\\&=W(\sin\alpha+\mu\cos\alpha)\end{aligned}$$

(iii) F will be minimum if the denominator on the right-hand side is maximum,

i.e., $\sin[\theta-(\alpha+\varphi)]=1$

or $\theta-\alpha+\varphi=90^\circ$

or $\theta-(90^\circ+\alpha)=\varphi$

i.e., the angle between F and the inclined plane should be equal to the angle of friction. In that case,

$$F_{min}=W\sin(\alpha+\varphi)$$

Efficiency The efficiency of an inclined plane, when a body slides up the plane, is defined as the ratio of the forces required to move the body without consideration and with consideration of force of friction.

Let F_o = force required to move the body up the plane without friction.

In the absence of friction, the force **R** coincides $\mathbf{R}_n$ and φ is zero,

and

$$F_o=\frac{W\sin\alpha}{\sin(\theta-\alpha)} \qquad \text{[Inserting } \phi=0 \text{ in Eq. (8.3)]} \qquad (8.4)$$

$$\begin{aligned}\eta&=\frac{F_o}{F}=\frac{W\sin\alpha}{\sin(\theta-\alpha)}\,\frac{\sin[\theta-(\alpha+\varphi)]}{W\sin(\alpha+\varphi)}\\&=\frac{\sin\alpha}{\sin(\alpha+\varphi)}\,\frac{\sin[\theta-(\alpha+\varphi)]}{\sin(\theta-\alpha)}\\&=\frac{\sin\alpha}{\sin(\alpha+\varphi)}\,\frac{\sin\theta\cos(\alpha+\varphi)-\cos\theta\sin(\alpha+\varphi)}{\sin\theta\cos\alpha-\cos\theta\sin\alpha}\end{aligned}$$

$$= \frac{\dfrac{\sin\alpha}{\sin(\alpha+\theta)}}{} \frac{\sin\theta\sin(\alpha+\theta)\left[\dfrac{\cos(\alpha+\varphi)}{\sin(\alpha+\varphi)}-\dfrac{\cos\theta}{\sin\theta}\right]}{\sin\theta\sin\alpha\left[\dfrac{\cos\alpha}{\sin\alpha}-\dfrac{\cos\theta}{\sin\theta}\right]}$$

$$= \frac{\cot(\alpha+\varphi)-\cot\theta}{\cot\alpha-\cot\theta} \tag{8.5}$$

If $\theta = 90°$, i.e., if the direction of the applied force is horizontal,

$$\eta = \frac{\cot(\alpha+\varphi)-\cot 90°}{\cot\alpha-\cot 90°}$$

$$= \frac{\cot(\alpha+\varphi)}{\cot\alpha}$$

$$= \frac{\tan\alpha}{\tan(\alpha+\varphi)} \tag{8.6}$$

3. Motion Down the Plane

When the body moves down the plane, the force of friction $\mathbf{F}'$ $(=\mu\mathbf{R}_n)$ acts in the upwards direction and the reaction **R**, i.e., the combination of $\mathbf{R}_n$ and $\mathbf{F}'$ is inclined backwards as shown in Fig. 8.5(a). Assume that **F** acts downwards.

Applying Lami's theorem as before [Fig. 8.5(b)],

$$\frac{F}{\sin[\pi-(\varphi-\alpha)]} = \frac{W}{\sin[\theta+(\varphi-\alpha)]}$$

$$F = \frac{W\sin(\varphi-\alpha)}{\sin[\theta+(\varphi-\alpha)]} \tag{8.7}$$

The equation suggests that F is positive only for $\varphi > \alpha$ and when $\varphi = \alpha$, the force required to slide the body down is zero, i.e., the body is on the point of moving down under its own weight W.

When $\varphi < \alpha$, i.e., the angle of friction is lesser than the angle of the inclined plane, F will be negative meaning that a force equal to F is to be applied in the opposite direction to resist the motion.

However, for a given value of α, F is minimum when the denominator of Eq. (8.7) is maximum

$$F_{\min} = W\sin(\varphi-\alpha) \tag{8.8}$$

If friction is neglected, i.e., $\varphi = 0$.

$$F_o = \frac{W\sin(-\alpha)}{\sin(\theta-\alpha)} = \frac{-W\sin\alpha}{\sin(\theta-\alpha)} \tag{8.9}$$

The force is negative indicating that in the absence of force of friction,

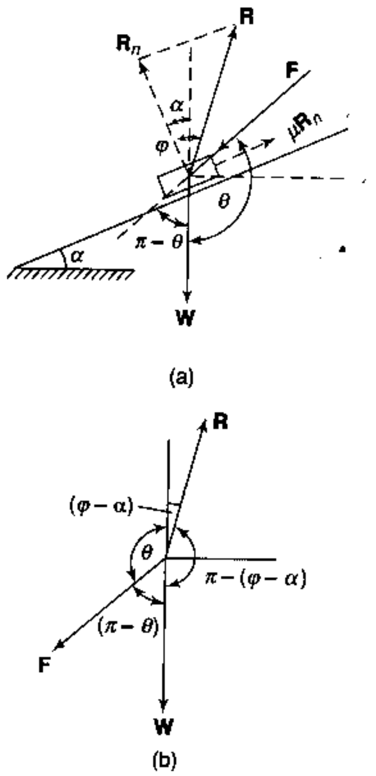


Fig. 8.5

a force in the opposite direction is required to oppose the motion down the plane. This is due to the fact that a component of **W** acts as an effort to move the body in the downward direction.

Efficiency Efficiency of the inclined plane when the body slides down the plane is defined as the ratio of the forces required to move the body with and without the consideration of force of friction, i.e.,

$$\eta = \frac{F}{F_o} = \frac{W\sin(\varphi-\alpha)}{\sin[\theta+(\varphi-\alpha)]}\,\frac{\sin(\theta-\alpha)}{W\sin\alpha}$$

$$= \frac{\sin(\varphi-\alpha)}{\sin\theta\cos(\varphi-\alpha)+\cos\theta\sin(\varphi-\alpha)}\,\frac{\sin\theta\cos\alpha-\cos\theta\sin\alpha}{\sin\alpha}$$

$$= \frac{\sin(\varphi-\alpha)}{\sin\theta\sin(\varphi-\alpha)\left[\dfrac{\cos(\varphi-\alpha)}{\sin(\varphi-\alpha)}+\dfrac{\cos\theta}{\sin\theta}\right]}\,\frac{\sin\theta\sin\alpha\left[\dfrac{\cos\alpha}{\sin\alpha}-\dfrac{\cos\theta}{\sin\theta}\right]}{\sin\alpha}$$

$$= \frac{\cot\alpha-\cot\theta}{\cot(\varphi-\alpha)+\cot\theta} \qquad (8.10)$$

When θ is 90° or the force applied is horizontal,

$$\eta = \frac{\cot\alpha}{\cot(\varphi-\alpha)} = \frac{\tan(\varphi-\alpha)}{\tan\alpha} \qquad (8.11)$$

Example 8.2 ***A body is to be moved up an inclined plane by applying a force parallel to the plane surface. It is found that a force of 3 kN is required to just move it up the plane when the angle of inclination is 10° whereas the force needed increases to 4 kN when the angle of inclination is increased to 15°. Determine the weight of the body and the coefficient of friction.***

Solution:
When the force applied is parallel to the plane surface,

$$F = W(\sin\alpha + \mu\cos\alpha)$$

$\therefore \quad 3000 = W(\sin 10° + \mu\cos 10°) \qquad$ (i)

and $\quad 4000 = W(\sin 15° + \mu\cos 15°)$

dividing (ii) by (i),

$$\frac{4000}{3000} = \frac{W(\sin 15° + \mu\cos 15°)}{W(\sin 10° + \mu\cos 10°)}$$

or $\quad (\sin 10° + \mu\cos 10°) = 0.75\,(\sin 15° + \mu\cos 15°)$

or $\quad \mu(\cos 10° - 0.75\cos 15°) = 0.75\sin 15° - \sin 10°$

or $\quad 0.2604\,\mu = 0.0205$

$\mu = \underline{0.0786}$

From (i) $3000 = W(\sin 10° + 0.0786\cos 10°)$

$W = 11\,950$ N or $\underline{11.95\text{ kN}}$

8.5 SCREW THREADS

A screw thread is obtained when the hypotenuse of a right-angled triangle is wrapped round the circumference of a cylinder.

Figure 8.6(a) shows a triangle *abc* which is the development of a helix of diameter d and lead l (or pitch p for a single start thread).

Length of base = circumference of the cylinder of screw threads

$$= \pi d$$

Height of triangle = l

$$\tan \alpha = \frac{l}{\pi d}$$

where α is the helix angle.

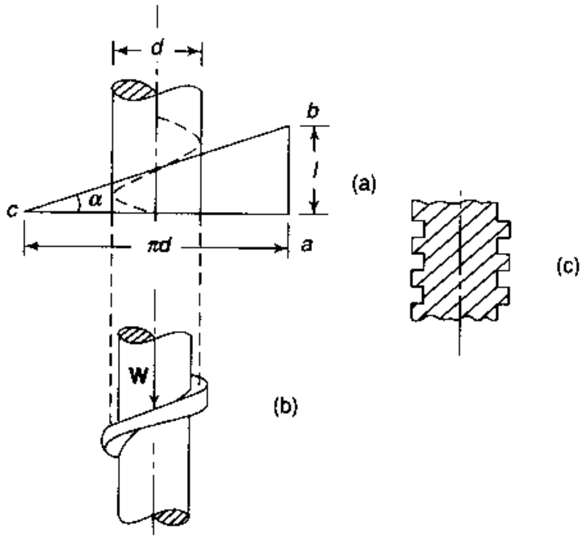


Fig. 8.6

Square Threads

A square-threaded screw used as a jack to raise a load W is shown in Fig 8.6(b). Faces of the square threads in the sectional view [Fig. 8.6(c)] are normal to the axis of the spindle. Force F acting horizontally is the force at the screw thread required to slide the load W up the inclined plane.

From Eq. (8.3) ($\theta = 90°$)

$$F = \frac{W \sin(\alpha + \varphi)}{\sin[90° - (\alpha + \varphi)]}$$

$$= \frac{W \sin(\alpha + \varphi)}{\cos(\alpha + \varphi)}$$

$$= W \tan(\alpha + \varphi) \qquad (8.12)$$

$$= W \frac{\tan \alpha + \tan \varphi}{1 - \tan \alpha \tan \varphi}$$

$$= W \frac{\frac{l}{\pi d} + \mu}{1 - \frac{l}{\pi d}\mu}$$

$$= W \frac{l + \mu \pi d}{\pi d - \mu l} \qquad (8.13)$$

A bar is usually fixed to the screw head to use as a lever for the application of force.
Let f = force applied at the end of the bar of length L
Then

$$fL = F\frac{d}{2} = Fr$$

or

$$f = \frac{Fr}{L} = \frac{Wr}{L}\tan(\alpha + \varphi) \qquad (8.14)$$

If the weight is to be lowered,

$$F = \frac{W \sin(\varphi - \alpha)}{\sin(90° + (\varphi - \alpha))} \quad \text{[from Eq. (8.7), } \theta = 90°\text{]}$$

$$= \frac{W \sin(\varphi - \alpha)}{\cos(\varphi - \alpha)}$$

$$= W \tan(\varphi - \alpha) \tag{8.15}$$

$$f = \frac{Wr}{L} \tan(\varphi - \alpha) \tag{8.15a}$$

$$\text{Screw efficiency} = \frac{\text{work done in lifting the load/rev.}}{\text{work done by the applied force/rev.}}$$

$$= \frac{W \times l}{F \times \pi d}$$

$$= \frac{W}{F} \times \frac{l}{\pi d}$$

$$= \frac{W}{W \tan(\alpha + \varphi)} \tan\alpha$$

$$= \frac{\tan\alpha}{\tan(\alpha + \varphi)} \tag{8.16}$$

This is maximum when $\dfrac{d\eta}{d\alpha} = 0$

$$\frac{d}{d\alpha}\left[\frac{\tan\alpha}{\tan(\alpha + \varphi)}\right] = 0$$

or

$$\frac{\sec^2\alpha \tan(\alpha + \varphi) - \sec^2(\alpha + \varphi)\tan\alpha}{\tan^2(\alpha + \varphi)} = 0$$

or

$$\sec^2\alpha \tan(\alpha + \varphi) - \sec^2(\alpha + \varphi)\tan\alpha = 0$$

or

$$\frac{\tan(\alpha + \varphi)}{\sec^2(\alpha + \varphi)} = \frac{\tan\alpha}{\sec^2\alpha}$$

or

$$\frac{\sin(\alpha + \varphi)}{\cos(\alpha + \varphi)} \cos^2(\alpha + \varphi) = \frac{\sin\alpha}{\cos\alpha} \cos^2\alpha$$

or

$$\sin(\alpha + \varphi)\cos(\alpha + \varphi) = \sin\alpha\cos\alpha$$

or

$$2\sin(\alpha + \varphi)\cos(\alpha + \varphi) = 2\sin\alpha\cos\alpha$$

or

$$\sin 2(\alpha + \varphi) = \sin 2\alpha$$

or

This is possible if either $(\alpha + \varphi) = \alpha$, i.e., $\varphi = 0$ (or no friction)

or $\quad \sin 2(\alpha + \varphi) = \sin(\pi - 2\alpha)$

i.e. $\quad 2(\alpha + \varphi) = \pi - 2\alpha$

$$4\alpha + 2\varphi = \pi$$

$$\alpha = \frac{\pi - 2\varphi}{4} = 45° - \frac{\varphi}{2}$$

Thus, the necessary condition for the maximum efficiency is

$$\alpha = 45° - \frac{\varphi}{2} \tag{8.17}$$

$$\text{Also, } \eta_{max} = \frac{\tan\left(45° - \dfrac{\varphi}{2}\right)}{\tan\left(45° - \dfrac{\varphi}{2} + \varphi\right)}$$

$$= \tan\left(45° - \frac{\varphi}{2}\right)\frac{1}{\tan\left(45° + \dfrac{\varphi}{2}\right)}$$

$$= \left(\frac{\tan 45° - \tan\dfrac{\varphi}{2}}{1 + \tan 45° \tan\dfrac{\varphi}{2}}\right)\left(\frac{1 - \tan 45° \tan\dfrac{\varphi}{2}}{\tan 45° + \tan\dfrac{\varphi}{2}}\right)$$

$$= \frac{\left(1 - \tan\dfrac{\varphi}{2}\right)\left(1 - \tan\dfrac{\varphi}{2}\right)}{\left(1 + \tan\dfrac{\varphi}{2}\right)\left(1 + \tan\dfrac{\varphi}{2}\right)}$$

$$= \frac{\left(1 - \tan\dfrac{\varphi}{2}\right)^2}{\left(1 + \tan\dfrac{\varphi}{2}\right)^2}$$

$$= \frac{\left(1 - \dfrac{\sin\varphi/2}{\cos\varphi/2}\right)^2}{\left(1 + \dfrac{\sin\varphi/2}{\cos\varphi/2}\right)^2}$$

$$= \frac{\left(\cos\dfrac{\varphi}{2} - \sin\dfrac{\varphi}{2}\right)^2}{\left(\cos\dfrac{\varphi}{2} + \sin\dfrac{\varphi}{2}\right)^2}$$

$$= \frac{\cos^2\dfrac{\varphi}{2} + \sin^2\dfrac{\varphi}{2} - 2\cos\dfrac{\varphi}{2}\sin\dfrac{\varphi}{2}}{\cos^2\dfrac{\varphi}{2} + \sin^2\dfrac{\varphi}{2} + 2\sin\dfrac{\varphi}{2}\cos\dfrac{\varphi}{2}}$$

$$= \frac{1 - 2\sin\frac{\varphi}{2}\cos\frac{\varphi}{2}}{1 + 2\sin\frac{\varphi}{2}\cos\frac{\varphi}{2}}$$

$$= \frac{1 - \sin\varphi}{1 + \sin\varphi} \qquad (8.18)$$

$$\text{Mechanical advantage} = \frac{\text{weight lifted}}{\text{force applied}} = \frac{W}{f} = \frac{W}{\frac{Wr}{L}\tan(\alpha + \varphi)}$$

$$= \frac{L}{r}\cot(\alpha + \varphi) \qquad (8.19)$$

$$\text{Velocity ratio} = \frac{\text{distance moved by force/rev.}}{\text{distance moved by load/rev.}}$$

or

$$VR = \frac{2\pi L}{l} = \frac{L}{\frac{l}{\pi d}\frac{d}{2}} = \frac{L}{r\tan\alpha} \qquad (8.20)$$

Observe that the angle of friction should always be more than the helix angle of the screw. Otherwise, the load will slide down of its own under the weight W. Such a condition is known as *overhauling of screws*. Thus, α is not to be more than φ to prevent the nut from turning back. Such a screw is known as *self-locking screw*. When $\alpha = \varphi$, the nut will be on the point of reversing and

$$\text{screw efficiency} = \frac{\tan\alpha}{\tan(\alpha + \varphi)} = \frac{\tan\varphi}{\tan 2\varphi} \approx \frac{1}{2} \qquad (8.21)$$

Thus, reversal of the nut is avoided if the efficiency of the thread is less than 50% (approximately).

Note that in case of square threads, as the helix angle α of screw threads is usually very small (3° – 8°), the faces of the threads are normal to the axis of spindle and thus, the normal reaction $\mathbf{R}_n$ is almost equal to the load $\mathbf{W}$ ($\cos\alpha \approx 1$).

V-Threads

In case of V-threads, the faces are inclined to the axis of the spindle even if the helix angle is neglected. Figure 8.7 shows a section through a V-thread, in which 2β is the angle between the faces of the thread (α has not been considered Thus, not shown). If $\mathbf{R}_n$ is the normal reaction then clearly the axial component of $\mathbf{R}_n$ must be equal to $\mathbf{W}$, i.e.,

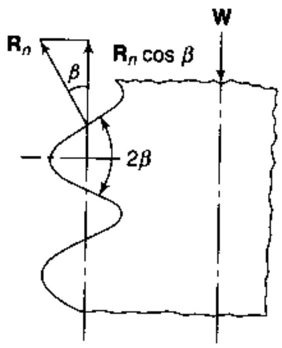


Fig. 8.7

$$W = R_n \cos\beta$$

or

$$R_n = \frac{W}{\cos\beta}$$

Friction force on the surface $= \mu R_n$

$$= \mu\frac{W}{\cos\beta}$$

$$= \frac{\mu}{\cos \beta} W$$

$$= \mu' W \qquad (8.22)$$

This shows that the coefficient of friction μ (or tan φ) as used in relations for the square threads is to be replaced by μ' or $\mu/\cos \beta$ or tan $\varphi/\cos \beta$ to adapt them to V-threads.

Example 8.3 *A square-threaded bolt with a core diameter of 25 mm and a pitch of 10 mm is tightened by screwing a nut. The mean diameter of the bearing surface of the nut is 60 mm. The coefficient of friction for the nut and the bolt is 0.12 and for the nut and the bearing surface, it is 0.15. Determine the force required at the end of a 400-mm long spanner if the load on the bolt is 12 kN.*

Solution:

The mean diameter of the threaded bolt = 25 + (10/2) = 30 mm

$$\text{Now, } \tan \alpha = \frac{p}{\pi d} = \frac{6}{\pi \times 30} = 0.0636, \ \alpha = 3.64^\circ$$

$$\tan \phi = 0.12 \qquad \phi = 6.84^\circ$$

$$T = Fr = W \tan(\alpha + \varphi)\, r$$

$$= 12\,000 \tan(3.64^\circ + 6.84^\circ) \times (28/2)$$

$$= 31\,076 \text{ N.m}$$

Torque due to friction between nut and bearing surface = $(\mu W)\, r$

$$= 0.15 \times 12\,000 \times 30$$

$$= 54\,000 \text{ N.mm}$$

Total friction torque required = 31076 - 54000 = 85076 N.mm

If F' is the force to be applied at the end of spanner,

$$F' \times l = T$$

$$F' \times 400 = 85076$$

$$F' = \underline{212.7 \text{ N}}$$

Example 8.4 *The cutting speed of a broaching machine is 9 m per minute. The cutter of the machine is pulled by a square-threaded screw with a nominal diameter of 60 mm and a pitch 12-mm. The operating nut takes an axial load of 500 N on a flat surface of 80 mm external diameter and 48-mm internal diameter. Determine the power required to rotate the operating nut. Take μ = 0.14 for all contact surfaces on the nut.*

Solution: The mean diameter of the threaded bolt = 60 - (12/2) = 54 mm

$$\text{Now, } \tan \alpha = \frac{p}{\pi d} = \frac{12}{\pi \times 54} = 0.0707, \ \alpha = 4.046^\circ$$

$$\mu = \tan \phi = 0.14, \qquad \phi = 7.97^\circ$$

$$T = Fr = W \tan(\alpha + \varphi)\, r$$

$$= 500 \tan(4.05^\circ + 7.97^\circ) \times (54/2)$$

$$= 2873 \text{ N.mm}$$

Mean radius of the flat surface = $(r_1 + r_2)/2$ = (40 + 24)/2 = 32 mm

Torque due to friction between nut and bearing surface = $(\mu W)\, r$

$$= 0.14 \times 500 \times 32$$

$$= 2240 \text{ N.mm}$$

Total friction torque required = 2873 + 2240 = 5113 N.mm or 5.113 N.m

As the threaded screw advances a distance equal to one pitch in one revolution,

The cutting speed $= p \times N$

or 9 000 $= 12 \times N$

or N = 750 rpm

∴ Power required to operate the nut = $T \times \omega$ = $5.113 \times \frac{2\pi \times 750}{60} = \underline{401.6 \text{ W}}$

Example 8.5 *Two railway coaches are coupled with the help of two tie rods of a turn buckle with right and left handed-threads having single-start square threads. The pitch and mean diameter of the threads are 8 mm and 30 mm respectively. What will be the work done in bringing the two coaches* *closer through a distance of 160 mm against a steady load of 2 kN? Take $\mu = 0.12$.*

Solution: Figure 8.8 shows the outline of a turn buckle.

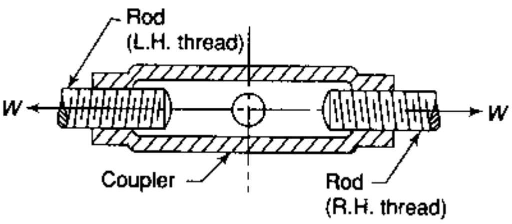


Fig. 8.8

$p = 8$ mm $\quad \mu = 0.12$

$d = 30$ mm $\quad W = 2000$ N

$$\tan\alpha = \frac{p}{\pi d} = \frac{8}{\pi \times 30} = 0.0848 \text{ or } \alpha = 4.85°$$

$\mu = \tan\varphi = 0.12$ or $\varphi = 6.84°$

Torque on each rod $= F.r = W\tan(\alpha + \varphi) \times r$

$= 2000 \times \tan(4.85° + 6.84°) \times 0.015$

$= 6.21$ N.m

Total torque on the coupling nut $= 2 \times 6.21 = 12.42$ N.m

In one complete revolution of the rod, each coach is moved through a distance equal to the pitch.

Number of turns required to move the coaches through a distance of 160 mm

$= 160/(2 \times 8) = 10$

Work done, $W = T.\theta = 12.42 \times 2\pi \times 10 = 780.4$ N.m

$$\eta = \frac{\tan\alpha}{\tan(\alpha + \varphi)} = \frac{\tan 4.85°}{\tan(4.85° + 6.84°)}$$

$= 0.41$ or 41%

Example 8.6 ***A whitworth bolt with an angle of V-threads as 55° has a pitch of 6 mm and a mean diameter of 32 mm. The mean radius of the bearing surface where the nut is tightened is 20 mm. Determine the force required at the end of a 400-mm long spanner when the load on the bolt is 8 kN. The coefficient of friction for the nut and the bolt is 0.1 and for the nut and the bearing surface is 0.15.***

Solution:

Virtual coefficient of friction,

$$\mu' = \frac{\mu}{\cos\beta} = \frac{0.1}{\cos 27.5°} = 0.113$$

or $\quad \tan\varphi = 0.113$ or $\varphi = 6.45°$

$$\tan\alpha = \frac{p}{\pi d} = \frac{6}{\pi \times 32} = 0.0597, \ \alpha = 3.42°$$

Torque transmitted $= F.r = W\tan(\alpha + \varphi) \times r$

$= 8000 \times \tan(3.42° + 6.45°) \times 16$

$= 22\,271$ N.mm

Torque due to friction between nut and bearing surface $= (\mu W)\, r$

$= 0.15 \times 8000 \times 20$

$= 24\,000$ N.mm

Total friction torque required $= 22\,271 + 24\,000 = 46\,271$ N.mm

If F' is the force to be applied at the end of spanner,

$F' \times l = T$

$F' \times 400 = 46\,271$

$F' = \underline{115.7 \text{ N}}$

8.6 SCREW JACK

A screw jack is a device used to lift heavy loads by applying a smaller effort at its handle. Figure 8.9 shows a common type of screw jack. It consists of a threaded screw that fits into the inner threads of the nut. The load is placed on the head of the threaded screw which is rotated by applying an effort at the end of a lever for lifting or lowering the load. The load placed on the head may rotate with the screw or it may be put on a swivel head (bearing) and thus, may not rotate with the screw. In that case, friction between the swivel head and the screw rod is also considered.

Expressions for the torque applied by the screw jack and its efficiency are given below:

Torque required to lift the load, $T = Fr = W\tan(\alpha + \varphi)\, r$

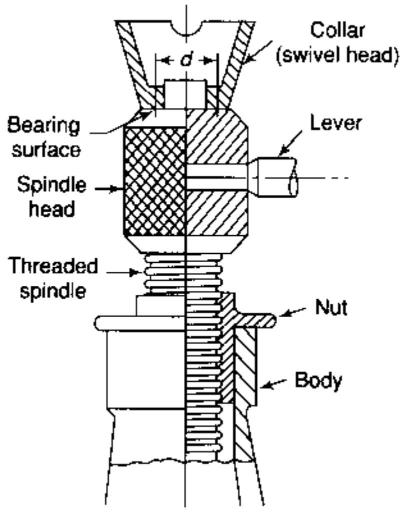


Fig. 8.9

Torque required to lower the load, $T = Fr = W \tan(\varphi - \alpha)\, r$

Mechanical advantage $= \dfrac{W}{P}$

Efficiency, $\eta = \dfrac{\tan \alpha}{\tan(\alpha + \varphi)}$

Example 8.7 ***A load of 15 kN is raised by means of a screw jack. The mean diameter of the square threaded screw is 42 mm and the pitch is 10 mm. A force of 120 N is applied at the end of a lever to raise the load. Determine the length of the lever to be used and the mechanical advantage obtained. Is the screw self-locking? Take $\mu = 0.12$.***

Solution:

$$\tan \alpha = \frac{p}{\pi d} = \frac{10}{\pi \times 42} = 0.0758, \quad \alpha = 4.335^\circ$$

$$\mu = \tan \phi = 0.12, \ \phi = 6.843^\circ$$

$$T = Fr = W \tan(\alpha + \varphi)\, r$$
$$= 15\,000 \tan(4.335^\circ + 6.843^\circ) \times (42/2)$$
$$= 62\,244 \text{ N.m}$$

Let l is be the force to be applied at the end of lever,

$$F \times l = T$$
$$120 \times l = 62\,244$$
$$l = \underline{518.7 \text{ mm}}$$

$$\text{MA} = \frac{W}{P} = \frac{15\,000}{120} = \underline{125}$$

Efficiency,

$$\eta = \frac{\tan \alpha}{\tan(\alpha + \varphi)} = \frac{\tan 4.335^\circ}{\tan(4.335^\circ + 6.843^\circ)} = 0.384$$

As the efficiency is less than 50%, therefore, the screw is self-locking.

Example 8.8 *The following data relate to a screw jack:*

Pitch of the threaded screw = 8 mm

Diameter of the threaded screw = 40 mm

Coefficient of friction between screw and nut = 0.1

Load = 20 kN

Assuming that the load rotates with the screw, determine the

(i) ratio of torques required to raise and lower the load

(ii) efficiency of the machine.

Solution:

$$\tan\alpha = \frac{P}{\pi d} = \frac{8}{\pi \times 40} = 0.0637$$

$\alpha = 3.64°$

$\mu = \tan\varphi = 0.1$ or $\phi = 5.71°$

(i) *To raise the load*

$T = Fr = W\tan(\alpha + \varphi)\,r$

$= 20\,000 \times \tan(3.64° + 5.71°) \times 0.02$

or $= 65.86$ N.m

To lower the load

$T = W\tan(\varphi - \alpha)r$

$= 20\,000 \times \tan(5.71° - 3.64°) \times 0.02$

$= 14.46$ N.m

$$\frac{\text{Torque to raise the load}}{\text{Torque to lower the load}} = \frac{65.86}{14.46} = \underline{4.56}$$

(ii) $$\text{Efficiency} = \frac{\tan\alpha}{\tan(\alpha + \varphi)}$$

$$= \frac{\tan 3.64°}{\tan(3.64° + 5.71°)} = 0.386 \text{ or } \underline{38.6\%}$$

Example 8.9 *In a screw jack, the diameter of the threaded screw is 40 mm and the pitch is 8 mm. The load is 20 kN and it does not rotate with the screw but is carried on a swivel head having a bearing diameter of 70 mm. The coefficient of friction between the swivel head and the spindle is 0.08 and between the screw and nut is 0.1. Determine the total torque required to raise the load and the efficiency.*

Solution:

$$\tan\alpha = \frac{P}{\pi d} = \frac{8}{\pi \times 40} = 0.0637$$

or $\alpha = 3.64°$

$\mu = \tan\varphi = 0.1$ or $\phi = 5.71°$

To raise the load,

$T = Fr = W\tan(\alpha + \varphi)\,r$

$= 20\,000 \times \tan(3.64° + 5.71°) \times 0.02$

or $= 65.86$ N.m

Torque due to collar friction $= (\mu W)r$

$= 0.08 \times 20\,000 \times 0.035$

$= 56$ N.m

Total friction torque required to raise the load

$= 65.86 + 56 = 121.86$ N.m

$$\eta = \frac{\text{Work done in lifting the load/rev.}}{\text{Work done by the applied force/rev.}}$$

$$= \frac{W \times p}{F \times \pi d} = \frac{W}{F}\tan\alpha$$

where $F = \dfrac{T}{r} = \dfrac{121.86}{0.02} = 6093$ N

$$\eta = \frac{20\,000}{6093} \times 0.0637 = 0.209 \text{ or } \underline{20.9\%}$$

Example 8.10 *A screw jack raises a load of 16 kN through a distance of 150 mm. The mean diameter and the pitch of the screw are 56 mm and 10 mm respectively. Determine the work done and the efficiency of the screw jack when the*

1. *load rotates with the screw*
2. *loose head on which the load rests does not rotate with the screw and the outside and the inside diameters of the bearing surface of the loose head are 50 mm and 10 mm respectively.*

Take coefficient of friction for the screw and the bearing surface as 0.11.

Solution:

$h = 150$ mm; $W = 16 \times 10^3$ N; $p = 10$ mm; $d = 56$ mm; $\mu = 0.11$

$$\tan\alpha = \frac{p}{\pi d} = \frac{10}{\pi \times 56} = 0.0568$$

or $\alpha = 3.25°$

$\mu = \tan\varphi = 0.11$ or $\phi = 6.28°$

To raise the load,

$T = Fr = W\tan(\alpha + \varphi)\,r$

$= 16\,000 \times \tan(3.25° + 6.28°) \times 0.028$

$= 75.211$ N.m

In one complete revolution, the distance moved by the screw is equal to one pitch or 10 mm.

$\therefore$ number of revolutions made by the screw = 150/10 = 15

(a) *When load rotates with the screw*

Work done in raising the load/rev. $= T.\,2\pi$

$\therefore$ total work done in raising the load

$= T.\,2\,\pi N = 75.211 \times 2\pi \times 15 = \underline{7088 \text{ N.m}}$

$$\text{Efficiency} = \frac{\tan\alpha}{\tan(\alpha+\varphi)} = \frac{\tan 3.25°}{\tan(3.25° + 6.28°)}$$
$$= 0.338 \text{ or } \underline{33.8\%}$$

- Efficiency can also be found from

$$\eta = \frac{\text{Work done in lifting the load/rev.}}{\text{Work done by the applied force/rev.}}$$
$$= \frac{W \times p}{F \times \pi d} = \frac{W}{F}\tan\alpha$$

where $F = \frac{T}{r} = \frac{75.211}{0.028} = 2686$ N

$$\eta = \frac{16\,000}{2686} \times 0.0568 = 0.338 \text{ or } \underline{33.8\%}$$

(a) *When load does not rotates with the screw*
Mean radius of the bearing surface,

$$r = \frac{1}{2}\left(\frac{50+10}{2}\right) = 15 \text{ mm}$$

Torque due to collar friction = $(\mu W)r$
$= 0.11 \times 16\,000 \times 0.015$
$= 26.4$ N.m

Total friction torque required to raise the load = 75.211 + 26.4 = 101.61 N.m

Work done in raising the load =
$T.2\pi N = 101.611 \times 2\pi \times 15 = \underline{9577 \text{ N.m}}$

$$\eta = \frac{\text{Work done in lifting the load/rev.}}{\text{Work done by the applied force/rev.}}$$
$$= \frac{W \times p}{F \times \pi d} = \frac{W}{F}\tan\alpha$$

where $F = \frac{T}{r} = \frac{101.61}{0.028} = 3629$ N

$$\eta = \frac{16\,000}{3629} \times 0.0568 = 0.25 \text{ or } \underline{25\%}$$

8.7 WEDGE

A wedge is used to raise loads like a screw jack. It consists of three sliding pairs as shown in Fig. 8.10(a) formed by the frame *A*, wedge *B* and the slider *S*. When a force **F** is applied to the wedge, the slider is raised in the guides raising the load.

Mechanical efficiency of the wedge is defined as the ratio of the load raised when friction is considered to the load raised when friction is neglected while the force applied is the same.

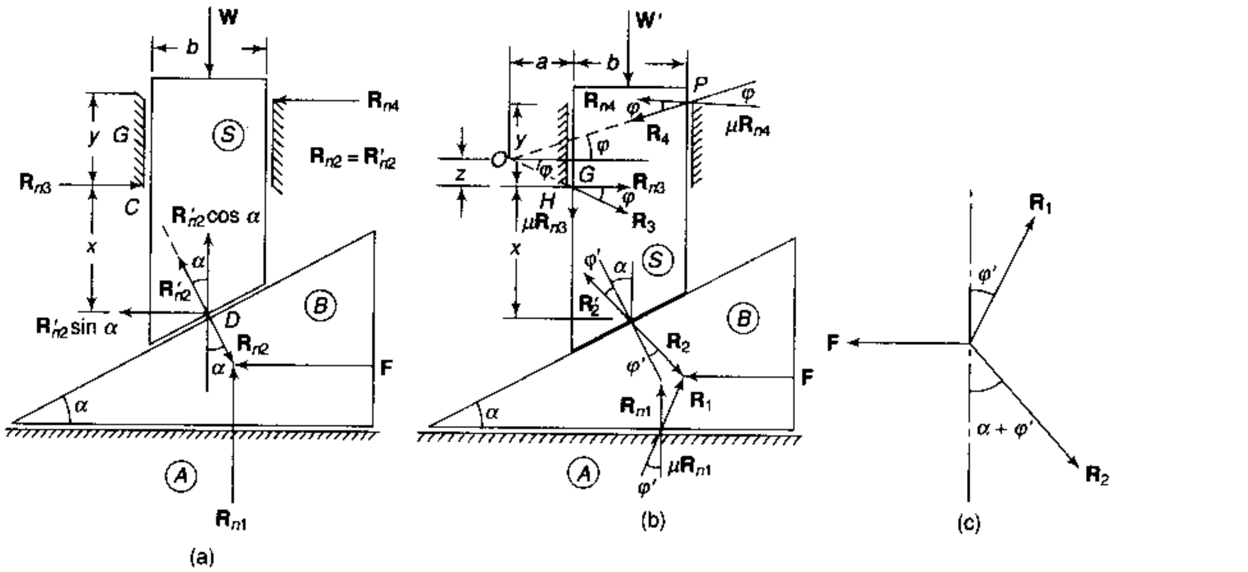


Fig. 8.10

1. Friction Neglected

(a) Equilibrium of Wedge The wedge is acted upon by [Fig. 8.10(a)] the
- horizontal force **F**
- reaction $\mathbf{R}_{n1}$ of the surface *A* in the vertical direction
- reaction $\mathbf{R}_{n2}$ of the slider, normal to slanting surface

For the equilibrium of the wedge, three forces must meet at a point and be balanced vectorially. Thus, $\mathbf{R}_{n2}$ passes through the point of intersection of **F** and $\mathbf{R}_{n1}$.

Also
$$F = R_{n2} \sin \alpha$$
$$R_{n1} = R_{n2} \cos \alpha = \frac{F}{\sin \alpha} \cos \alpha = F \cot \alpha$$

(b) Equilibrium of Slider The slider is acted upon by the
- weight **W**, vertically downwards
- reaction $\mathbf{R}'_{n2}$ of the wedge, equal and opposite to $\mathbf{R}_{n2}$
- reaction of the guide

The guide surface is vertical. Therefore, it exerts a horizontal reaction force on the slider. The three forces must also meet at a point, i.e., the reaction of the guide must also pass through *D*. However, in practice, the guide surface will be above *D* as shown in Fig. 8.10(a). Now, if it is assumed that the reaction of the guide acts through *C*, the lower end of the guide, and is equal to the horizontal component of $\mathbf{R}'_{n2}$ (= $\mathbf{R}'_{n2} \sin \alpha$), a clockwise couple will act on the slider. The slider is balanced only if two reaction forces $\mathbf{R}_{n3}$ at the lower end and $\mathbf{R}_{n4}$ on the upper end of the guide act as shown in the figure.

Let x = height of lower end of guide from *D*

y = height of the guide.

Balancing the horizontal and vertical components of forces,
$$R_{n3} - R_{n4} = R'_{n2} \sin \alpha = R_{n2} \sin \alpha = F$$
and
$$W = R'_{n2} \cos \alpha = R_{n2} \cos \alpha = R_{n1} = F \cot \alpha = \frac{F}{\tan \alpha} \qquad \text{(i)}$$

Taking moments about *D*,
$$R_{n3} \times x = R_{n4} \times (x + y) = R_{n4}\, x + R_{n4}\, y$$
or
$$(R_{n3} - R_{n4}) = \frac{R_{n4}\, y}{x}$$
or
$$F = \frac{R_{n4}\, y}{x}$$
or
$$R_{n4} = \frac{Fx}{y}$$
and
$$R_{n3} = F + F\frac{x}{y} = F\left(1 + \frac{x}{y}\right)$$

2. Friction Considered

Let φ' = Angle of friction between frame *A* slider *S* and wedge *B*

φ = Angle of friction between guide *G* and slider *S*

(a) Equilibrium of Wedge The forces acting on the wedge are [Fig.8.10(b)] the

- horizontal force **F**
- reaction $\mathbf{R}_1$ inclined at an angle φ' with $\mathbf{R}_{n1}$. the wedge moves towards left and so the friction force acts towards right
- reaction $\mathbf{R}_2$ inclined at an angle φ' with $\mathbf{R}_{n2}$.

For equilibrium [Fig. 8.10(c)],

$$\frac{R_1}{\sin[90° + (\alpha + \varphi')]} = \frac{R_2}{\sin(90° + \varphi')} = \frac{F}{\sin[180° - (\alpha + 2\varphi')]}$$

$$\frac{R_1}{\cos(\alpha + \varphi')} = \frac{R_2}{\cos\varphi'} = \frac{F}{\sin(\alpha + 2\varphi')}$$

$$R_1 = \frac{\cos(\alpha + \varphi')}{\sin(\alpha + 2\varphi')} F$$

and

$$R_2 = \frac{\cos\varphi'}{\sin(\alpha + 2\varphi')} F$$

(b) Equilibrium of Slider The slider is acted upon by the

- reaction $\mathbf{R}'_2$ of the wedge, equal and opposite to $\mathbf{R}_2$
- weight $\mathbf{W}'$, vertically downwards
- reactions $\mathbf{R}_3$ and $\mathbf{R}_4$ of the guide

The motion of the guide is vertically upwards. Therefore, the friction force acts in the downward direction.

Let $\mathbf{R}_3$ and $\mathbf{R}_4$ intersect at O. the distances a and z are as shows in the diagram.

Taking moments about O,

$$R'_2[\sin(\alpha + \varphi')](x + z) + W'\left(a + \frac{b}{2}\right) - R'_2\cos(\alpha + \varphi')\left(a + \frac{b}{2}\right) = 0$$

$$W' = R'_2\left[\cos(\alpha + \varphi') - \frac{(x + z)\sin(\alpha + \varphi')}{\left(a + \frac{b}{2}\right)}\right]$$

$$= \frac{F\cos\varphi'}{\sin(\alpha + 2\varphi')}\left[\cos(\alpha + \varphi') - \frac{(x + z)\sin(\alpha + \varphi')}{\left(a + \frac{b}{2}\right)}\right] \quad \text{(ii)}$$

Dividing (ii) by (i),

$$\eta = \frac{W'}{W} = \frac{\cos\varphi'\tan\alpha}{\sin(\alpha + 2\varphi')}\left[\cos(\alpha + \varphi') - \frac{(x + z)\sin(\alpha + \varphi')}{\left(a + \frac{b}{2}\right)}\right]$$

If x and b are comparatively small and can be neglected,

$$\eta = \frac{\cos\varphi'\tan\alpha}{\sin(\alpha + 2\varphi')}\left[\cos(\alpha + \varphi') - \frac{z}{a}\sin(\alpha + \varphi')\right]$$

$$= \frac{\cos\varphi' \tan\alpha}{\sin(\alpha + 2\varphi')} [\cos(\alpha + \varphi') - \tan\varphi \sin(\alpha + \varphi')] \quad (8.23)$$

$$= \frac{\cos\varphi' \tan\alpha}{\sin(\alpha + 2\varphi')} \left[\frac{\cos(\alpha + \varphi')\cos\varphi - \sin\varphi \sin(\alpha + \varphi')}{\cos\varphi}\right]$$

$$= \frac{\cos\varphi' \tan\alpha}{\sin(\alpha + 2\varphi')} \times \frac{\cos(\alpha + \varphi + \varphi')}{\cos\varphi} \quad (8.24)$$

If $\varphi = \varphi'$,

$$\eta = \frac{\tan\alpha}{\tan(\alpha + 2\varphi)} \quad (8.25)$$

Example 8.11 *Determine the efficiency of the wedge shown in Fig. 8.10(a). The angle of the wedge is 30° and coefficient of friction for the wedge and the slider is 8° and for the guide and the slider, it is 6°. Determine the efficiency.*

Solution:

$\mu = \tan\varphi = \tan 6° = 0.105$

From Eq. (8.24),

$$\eta = \frac{\cos\varphi' \tan\alpha}{\sin(\alpha + 2\varphi')} \times \frac{\cos(\alpha + \varphi + \varphi')}{\cos\varphi}$$

$$= \frac{\cos 8° \tan 30°}{\sin(30° + 16°)} \frac{\cos(30° + 6° + 8°)}{\cos 6°}$$

$= 0.547$

8.8 PIVOTS AND COLLARS

When a rotating shaft is subjected to an axial load, the thrust (axial force) is taken either by a pivot or a collar. Examples are the shaft of a steam turbine and propeller shaft of a ship.

Collar Bearing

A collar bearing or simply a collar is provided at any position along the shaft and bears the axial load on a mating surface.

The surface of the collar may be plane (flat) normal to the shaft (Fig. 8.11) or of conical shape (Fig. 8.12).

Pivot Bearing

When the axial load is taken by the end of the shaft which is inserted in a recess to bear the thrust, it is called a *pivot bearing* or simply a *pivot*. It is also known as *footstep bearing*.

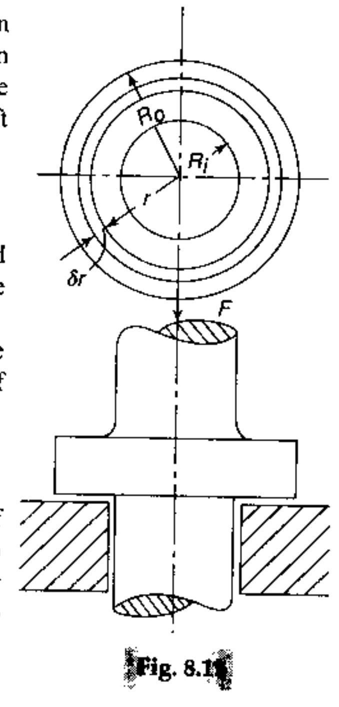


Fig. 8.11

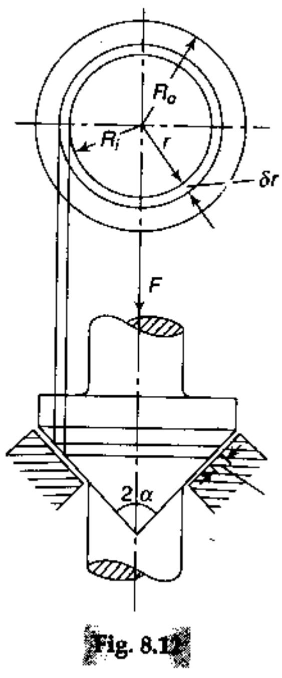


Fig. 8.12

The surface of the pivot can also be flat or of conical shape [Figs. 8.13(a) and (b)].

Uniform Pressure and Uniform Wear

Friction torque of a collar or a pivot bearing is calculated, usually, on the basis of two assumptions. Each assumption leads to a different value of torque. In one case, it is assumed that the intensity of pressure on the bearing surface is constant whereas in the second case, it is the uniform wearing of the bearing surface.

Under the first assumption, pressure is assumed to be uniform over the surface area and the intensity of pressure is given by (Fig. 8.11).

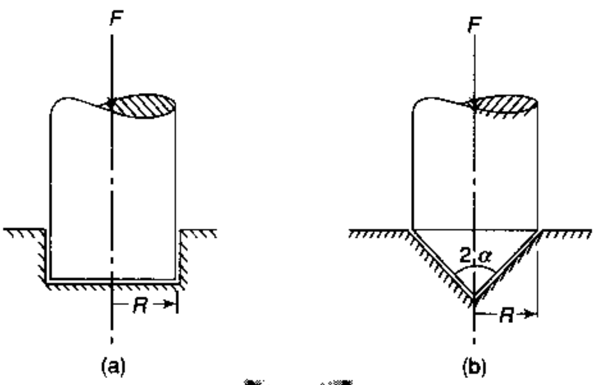


Fig. 8.13

$$\text{Pressure} = \frac{\text{axial force}}{\text{cross-sectional area}}$$

or

$$p = \frac{F}{\pi (R_0^2 - R_i^2)} \quad (8.26)$$

where R_o = outer radius of the collar
R_i = inner radius of the collar

For uniform wear over an area, the intensity of pressure should vary inversely proportional to the elementary areas, i.e., it should decrease with the increase in the elementary area and vice-versa. This can be illustrated by drawing a line with a chalk. In doing so, a little quantity of chalk is worn from the stick. Now, if it is desired that the chalk is worn by the same amount, but the length of the line is doubled, the pressure on the chalk has to be reduced to half that in the previous case. Therefore, for uniform wear, product of the pressure applied and the distance travelled must be constant. For uniform wear of the surface, let

p_1 = normal pressure between two surfaces at radius r_1
p_2 = normal pressure between two surfaces at radius r_2
b = width of the surface at radii r_1 and r_2 (equal width)

$$p_1 \times \text{area at } r_1 = p_2 \times \text{area at } r_2$$

$$p_1 \times 2\,\pi r_1 \times b = p_2 \times 2\,\pi r_2 \times b$$

or $$p_1 r_1 = p_2 r_2$$

or $$pr = \text{constant} \quad (8.27)$$

Thus, in case of uniform weariness of the two surfaces, product of the normal pressure and the corresponding radius must be constant. This means the pressure is less where the radius is more and vice-versa. Pressure on an elemental area at radius r can be found as given below.

$$\text{Axial force, } F = \int_{R_i}^{R_o} \text{Axial force on the elemental area}$$

$$= \int_{R_i}^{R_o} \text{Pressure on the element} \times \text{Area}$$

$$= \int_{R_i}^{R_o} p \times 2\pi r dr$$

$$= \int_{R_i}^{R_o} \frac{C}{r} \times 2\pi r dr \qquad (p.r = C)$$

$$= \int_{R_i}^{R_o} 2\pi C.dr$$

$$= (2\pi Cr)_{R_i}^{R_o}$$

$$= 2\,\pi\, C\,(R_o - R_i)$$

$$= 2\,\pi\, pr\,(R_o - R_i)$$

or pressure intensity p at a radius r of the collar,

$$p = \frac{F}{2\pi r\,(R_o - R_i)} \qquad (8.28)$$

In a flat pivot, in which $R_i = 0$, the pressure would be infinity at the centre of the bearing ($r = 0$), which cannot be true. Thus, the uniform wear theory has a flaw in it.

Collars and pivots, using the above two theories, have been analysed below:

Collars

(i) Flat Collar

Let p = uniform normal pressure over an area
F = axial thrust
N = speed of the shaft
μ = coefficient of friction between the two surfaces

Consider an element of width δr of the collar at radius r. Friction force on the element (Fig. 8.10),

$$\delta F = \mu \times \text{axial force}$$
$$= \mu \times p \times \text{area of the element}$$
$$= \mu \times p \times 2\,\pi\, r\, \delta r$$

Friction torque about the shaft axis,

$$\delta T = \delta F \times r$$
$$= \mu \times p \times 2\,\pi\, r\, \delta r \times r$$
$$= 2\,\mu\, p\, \pi\, r^2\, \delta r$$

$$\text{Total friction torque, } T = \int_{R_i}^{R_o} 2\mu p \pi r^2 dr \qquad (8.29)$$

(a) *With Uniform Pressure Theory* Pressure is uniform over the whole area and is given by

$$p = \frac{F}{\pi(R_o^2 - R_i^2)}$$

$$T = \int_{R_i}^{R_o} 2\mu \pi r^2 \frac{F}{\pi\,(R_o^2 - R_i^2)} dr$$

$$= \int_{R_i}^{R_o} \frac{2\mu F}{R_o^2 - R_i^2} r^2 dr$$

$$= \left[\frac{2\mu F}{R_o^2 - R_i^2} \cdot \frac{r^3}{3} \right]_{R_i}^{R_o}$$

$$= \frac{2\mu F (R_o^3 - R_i^3)}{3(R_o^2 - R_i^2)} \tag{8.30}$$

(b) With Uniform Wear Theory Pressure p at a radius r of the collar is given by

$$p = \frac{F}{2\pi r (R_o - R_i)}$$

$$\therefore \quad T = \int_{R_i}^{R_o} 2\mu\pi r^2 \frac{F}{2\pi r (R_o - R_i)} dr$$

$$= \int_{R_i}^{R_o} \frac{\mu F r}{R_o - R_i} dr$$

$$= \frac{\mu F (R_o^2 - R_i^2)}{2(R_o - R_i)}$$

$$= \frac{\mu F}{2}(R_o + R_i)$$

$$= \mu F \times \text{Mean radius of the collar bearing} \tag{8.31}$$

(ii) Conical Collar (Frustum of Cone) This is also known as *trapezoidal* or *truncated conical pivot.* Consider an elementary area of width δr at a radius r of the bearing (Fig. 8.11).

$$\text{Normal force on the elementary area} = \frac{\text{Axial force}}{\sin\alpha}$$

$$\text{Normal pressure on the elementary area} = \frac{\text{Axial force}}{\sin\alpha} \cdot \frac{1}{\text{Surface area}}$$

$$= \frac{\text{Axial force}}{\sin\alpha} \cdot \frac{1}{2\pi r.\delta r / \sin\alpha}$$

$$= \frac{\text{Axial force}}{2\pi r.\delta r}$$

$$= \frac{\text{Axial force}}{\text{Area} \perp \text{ to axial force}}$$

$$= \text{Axial pressure } (p)$$

i.e., normal pressure on the surface is equal to the axial pressure on a flat collar surface.

Friction force on the element,

$$\delta F = \mu \times p \times \text{Area of the element}$$

$$= \mu \times p \times 2\pi r \frac{\delta r}{\sin\alpha}$$

Friction torque about the shaft axis,

$$\delta T = \delta F \times r = \frac{2\mu p \pi r^2}{\sin\alpha}\delta r$$

Total friction torque,

$$T = \int_{R_i}^{R_o} \frac{2\mu p \pi r^2}{\sin\alpha} dr \tag{8.32}$$

(a) With Uniform Pressure Theory

$$T = \int_{R_i}^{R_o} \frac{2\mu\pi r^2}{\sin\alpha} \frac{F}{\pi(R_o^2 - R_i^2)} dr$$

$$= \frac{2\mu F}{\sin\alpha(R_o^2 - R_i^2)}\left[\frac{r^3}{3}\right]_{R_i}^{R_o}$$

$$= \frac{2\mu F}{3\sin\alpha}\left[\frac{R_o^3 - R_i^3}{R_o^2 - R_i^2}\right] \tag{8.33}$$

i.e., the torque is increased in the ratio $\frac{1}{\sin\alpha}$ from that for flat collars.

(b) With Uniform Wear Theory

$$T = \int_{R_i}^{R_o} \frac{2\mu\pi r^2}{\sin\alpha} \frac{F}{2\pi r(R_o - R_i)} dr$$

$$= \int_{R_i}^{R_o} \frac{\mu F}{\sin\alpha(R_o - R_i)} r dr$$

$$= \frac{\mu F}{\sin\alpha(R_o - R_i)}\left(\frac{r^2}{2}\right)_{R_i}^{R_o}$$

$$= \frac{\mu F}{2\sin\alpha}(R_o + R_i) \tag{8.34}$$

$$= \frac{\mu F}{\sin\alpha} \times \text{mean radius of the bearing}$$

i.e., the torque is increased by $\frac{1}{\sin\alpha}$ times from that for flat collars.

Pivots

Expressions for torque in case of pivots can directly be obtained from the expressions for collars by inserting the values $R_i = 0$ and $R_o = R$.

(i) Flat Pivot

(a) Uniform pressure theory, $T = \frac{2}{3}\mu FR$ (8.35)

(b) Uniform wear theory, $T = \frac{1}{2}\mu FR$ (8.36)

(ii) Conical Pivot

(a) Uniform pressure theory, $T = \frac{2\mu FR}{3\sin\alpha}$ (8.37)

(b) Uniform wear theory, $T = \frac{\mu FR}{2\sin\alpha}$ (8.38)

The above expressions reveal that the value of the friction torque is more when the uniform pressure theory is applied. In practice, however, it has been found that the value of the friction torque lies in between that given by the two theories. To be on the safer side, out of the two theories, one is selected on the basis of the use.

A clutch plate transmits torque through the force of friction. Thus, though a clutch will surely be transmitting torque given by the uniform wear theory (lower value), it is not necessary that the clutch can also transmit a torque given by the uniform pressure theory (higher value). Therefore, it is safer to say that the clutch transmits a maximum torque based on the uniform wear theory and design it accordingly. However, the actual torque transmitted will be a little higher.

On the other hand, while calculating the power loss in a bearing, it is to be on the basis of uniform pressure theory, though the actual power loss will be a little less than that calculated.

Example 8.12 ***In a thrust bearing, the external and the internal diameters of the contacting surfaces are 320 mm and 200 mm respectively. The total axial load is 80 kN and the intensity of pressure is 350 kN/m². The shaft rotates at 400 rpm. Taking the coefficient of friction as 0.06, calculate the power lost in overcoming the friction. Also, find the number of collars required for the bearing.***

Solution:

$R_o = 0.16$ m $\quad F = 80 \times 10^3$ N

$R_i = 0.1$ m $\quad \mu = 0.06$

$N = 400$ rpm $\quad p = 350 \times 10^3$ N/m^2

Using uniform pressure theory as we are to find the power loss in a bearing,

$$T = \frac{2}{3}\mu F\left(\frac{R_o^3 - R_i^3}{R_o^2 - R_i^2}\right)$$

$$= \frac{2}{3} \times 0.06 \times 80 \times 10^3 \left[\frac{(0.16)^3 - (0.10)^3}{(0.16)^2 - (0.10)^2}\right]$$

$$= 3200 \times 0.1985$$

$$= 635.12 \text{ N.m}$$

$$P = T\omega = T\frac{2\pi N}{60} = 635.12 \times \frac{2\pi \times 400}{60}$$

$$= 26\,602 \text{ W or } 26.602 \text{ kW}$$

$$\text{Number of collars} = \frac{\text{total load}}{\text{load per collar}}$$

$$= \frac{F}{p \times \pi(R_o^2 - R_i^2)}$$

$$= \frac{80 \times 10^3}{350 \times 10^3 \times \pi[(0.16)^2 - (0.10)^2]}$$

=4.66 or 5 collars

Example 8.13 ***A conical pivot with angle of cone as 100° supports a load of 18 kN. The external radius is 2.5 times the internal radius. The shaft rotates at 150 rpm. If the intensity of pressure is to be 300 kN/m² and coefficient of friction as 0.05, what is the power lost in working against the friction?***

Solution:

$F = 18$ kN $\quad R_o = 2.5R_i$

$p = 300$ kN/m^2 $\quad N = 150$ rpm

$\mu = 0.05$ $\quad \alpha = 50°$

In case of uniform pressure, normal pressure

$$p = \frac{F}{\pi(R_o^2 - R_i^2)}$$

or $$300 \times 10^3 = \frac{18 \times 10^3}{\pi[(2.5R_i)^2 - R_i^2]}$$

or $$(2.5R_i)^2 - R_i^2 = \frac{18}{300 \times \pi}$$

$R_i = 0.0603$ m

$R_o = 0.0603 \times 2.5 = 0.1508$ m

$$T = \frac{2}{3}\frac{\mu F}{\sin\alpha}\left[\frac{R_o^3 - R_i^3}{R_o^2 - R_i^2}\right]$$

$$T = \frac{2}{3} \times \frac{0.05 \times 18000}{\sin 50°}$$

$$\left[\frac{(0.1508)^3 - (0.0603)^3}{(0.1508)^2 - (0.0603)^2}\right] = 131.6 \text{ N.m}$$

$$P = T\omega = T\frac{2\pi N}{60} = 131.6 \times \frac{2\pi \times 150}{60}$$

$= 2067$ W or $\underline{2.067 \text{kW}}$

Example 8.14 ***A thrust bearing of a propeller shaft consists of a number of collars. The shaft is of 400 mm diameter and rotates at a speed of 90 rpm. The thrust on the shaft is 300 kN. If the intensity of pressure is to be 200 kN/m² and coefficient of friction is 0.06, determine external diameter of the collars and the number of collars. The power lost in friction is not to exceed 48 kW.***

Solution:

$R_i = 200$ mm; $N = 90$ rpm; $F = 300 \times 10^3$ N;

$p = 200$ kN/m^2 $= 0.2$ N/mm^2

$P = 48$ kW; $\mu = 0.06$

$$P = T.\frac{2\pi N}{60} \text{ or } 48\,000 = T.\frac{2\pi \times 90}{60}$$

or $T = 5093$ N.m $= 5093 \times 10^3$ N.mm

Let R_o be the external radius of the collar.

$$T = \frac{2}{3}\mu F\left(\frac{R_o^3 - R_i^3}{R_o^2 - R_i^2}\right)$$

$$= \frac{2}{3}\mu F\left(\frac{(R_o - R_i)(R_o^2 + R_i^2 + R_oR_i)}{(R_o - R_i)(R_o + R_i)}\right)$$

$$= \frac{2}{3}\mu F\left(\frac{R_o^2 + R_i^2 + R_oR_i}{R_o + R_i}\right)$$

or $$5093 \times 10^3 = \frac{2}{3} \times 0.06 \times 300 \times 10^3 \left[\frac{R_o^2 + 200^2 + 200R_o}{R_o + 200}\right]$$

or $$424.4(R_o + 200) = R_o^2 + 200^2 + 200R_o$$

or $$R_o^2 - 224.4R_o - 44\,880 = 0$$

or $$R_o = \frac{224.4 \pm \sqrt{224.4^2 + 4 \times 44\,880}}{2}$$

$= 352$ mm (Taking positive sign only)

In case of uniform pressure, normal pressure

$$p = \frac{F}{\pi n(R_o^2 - R_i^2)}$$

or $$0.2 = \frac{300 \times 10^3}{\pi n[352^2 - 200^2]}$$

$n = 5.69$

Thus, number of collars = 6

8.9 FRICTION CLUTCHES

A clutch is a device used to transmit the rotary motion of one shaft to another when desired. The axes of the two shafts are coincident.

In friction clutches, the connection of the engine shaft to the gear-box shaft is affected by friction between two or more rotating concentric surfaces. The surfaces can be pressed firmly against one another when engaged and the clutch tends to rotate as a single unit.

1. Disc Clutch (Single-plate Clutch)

A disc clutch consists of a clutch plate attached to a splined hub which is free to slide axially on splines cut on the driven shaft. The clutch plate is made of steel and has a ring of friction lining on each side. The engine shaft supports a rigidly fixed flywheel.

A spring-loaded pressure plate presses the clutch plate firmly against the flywheel when the clutch is engaged. When disengaged, the springs press against a cover attached to the flywheel. Thus, both the flywheel and the pressure plate rotate with the input shaft. The movement of the clutch pedal is transferred to the pressure plate through a thrust bearing.

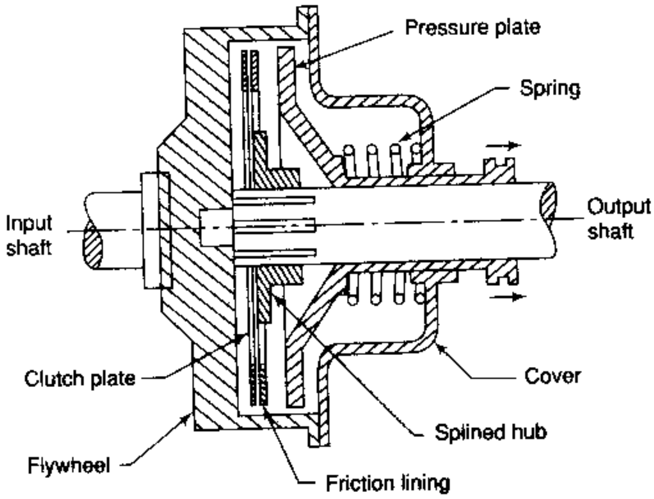


Fig. 8.14

Figure 8.14 shows the pressure plate pulled back by the release levers and the friction linings on the clutch plate are no longer in contact with the pressure plate or the flywheel. The flywheel rotates without driving the clutch plate and thus, the driven shaft.

When the foot is taken off the clutch pedal, the pressure on the thrust bearing is released. As a result, the springs become free to move the pressure plate to bring it in contact with the clutch plate. The clutch plate slides on the splined hub and is tightly gripped between the pressure plate and the flywheel. The friction between the linings on the clutch plate, and the flywheel on one side and the pressure plate on the other, cause the clutch plate and hence the driven shaft to rotate.

In case the resisting torque on the driven shaft exceeds the torque at the clutch, clutch slip will occur.

2. Multi-plate Clutch

In a multi-plate clutch, the number of frictional linings and the metal plates is increased which increases the capacity of the clutch to transmit torque. Figure 8.15 shows a simplified diagram of a multi-plate clutch.

The friction rings are splined on their outer circumference and engage with corresponding splines on the flywheel. They are free to slide axially. The friction material Thus, rotates with the flywheel and the engine shaft. The number of friction rings depends upon the torque to be transmitted.

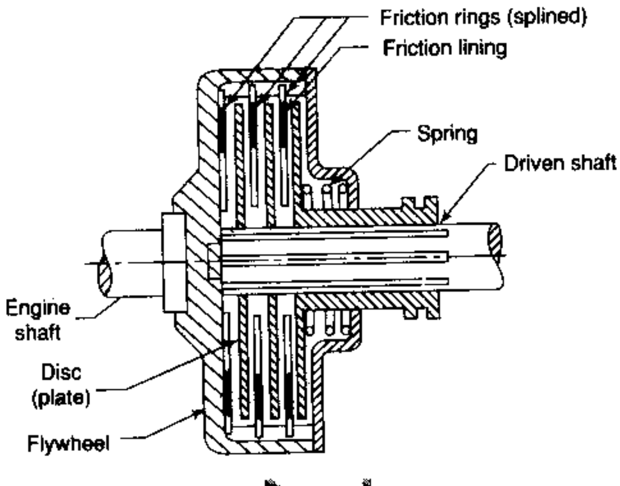


Fig. 8.15

The driven shaft also supports discs on the splines which rotate with the driven shaft and can slide axially. If the actuating force on the pedal is removed, a spring presses the discs into contact with the friction rings and the torque is transmitted between the engine shaft and the driven shaft.

If n is the total number of plates both on the driving and the driven members, the number of active surfaces will be $n-1$.

3. Cone Clutch

In a cone clutch (Fig. 8.16), the contact surfaces are in the form of cones. In the engaged position, the friction surfaces of the two cones A and B are in complete contact due to spring pressure that keeps one cone pressed against the other all the time.

When the clutch is engaged, the torque is transmitted from the driving shaft to the driven shaft through the flywheel and the friction cones. For disengaging the clutch, the cone B is pulled back through a lever system against the force of the spring.

The advantage of a cone clutch is that the normal force on the contact surfaces is increased. If F is the axial force, F_n the normal force and α the semi-cone angle of the clutch then for a conical collar with uniform wear theory,

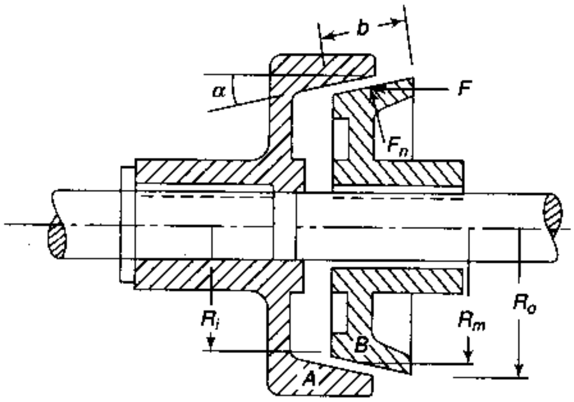


Fig. 8.16

$$F_n = \frac{F}{\sin\alpha}$$

$$= \frac{2\pi pr(R_o - R_i)}{\sin\alpha} \qquad \text{(Refer Eq. 8.28)}$$

$$= 2\pi pr.b \qquad \left(\sin\alpha = \frac{R_o - R_i}{b}\right) \qquad (8.39)$$

where b is the width of the cone face. Remember as pr is constant in case of uniform wear theory which is applicable to clutches to be on the safer side, p is to be the normal pressure at the radius considered, i.e., at the inner radius it is $p_i r_i$ and at the mean radius $p_m R_m$.

Also,

$$T = \frac{\mu F}{2\sin\alpha}(R_o + R_i) \qquad \text{(Eq. 8.34)}$$

$$= \frac{\mu F_n \sin\alpha}{\sin\alpha} \cdot \frac{(R_o + R_i)}{2}$$

$$= \mu F_n R_m \qquad (R_m = \text{Mean radius of the clutch})$$

However, cone clutches have become obsolete as small cone angles and exposure to dust and dirt tend to bind the two cones and it becomes difficult to disengage them.

4. Centrifugal Clutch

Centrifugal clutches are being increasingly used in automobiles and machines. A centrifugal clutch has a driving member consisting of four sliding blocks (Fig. 8.17). These blocks are kept in position by means of flat springs provided for the purpose. As the speed of the shaft increases, the centrifugal force on the shoes increases. When the centrifugal force exceeds the resisting force of the springs, the shoes move forward and press against the inside of the rim and thus the torque is transmitted to the rim. In this way, the clutch is engaged only when the motor gains sufficient speed to take up the load in an effective manner. The outer surfaces of the shoes are lined with some friction material.

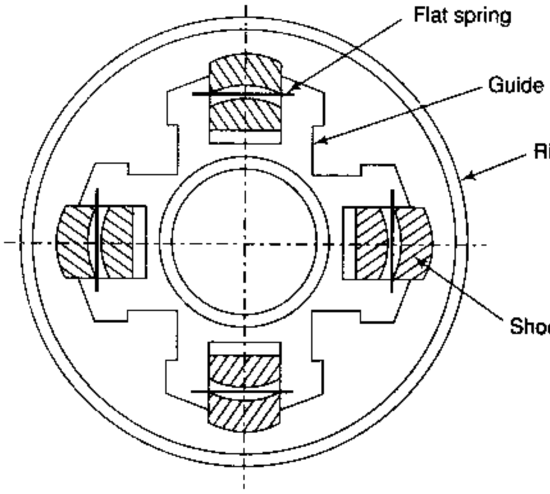


Fig. 8.17

Let

m = mass of each shoe
R = inner radius of the pulley rim
r = distance of centre of mass of each shoe from the shaft axis
n = number of shoes
ω = normal speed of the shaft in rad/s
ω' = Speed at which the shoe moves forward
μ = coefficient of friction between the shoe and the rim

Centrifugal force exerted by each shoe at the time of engagement with the rim = $mr\omega'^2$

This will be equal to the resisting force of the spring.

Centrifugal force exerted by each shoe at normal speed $= mr\omega^2$

Net normal force exerted by each shoe on the rim $= mr\omega^2 - mr\omega'^2$

$= mr(\omega^2 - \omega'^2)$

Frictional force acting tangentially on each shoe $= \mu\, mr(\omega^2 - \omega'^2)$

Frictional torque acting on each shoe $= \mu\, mr(\omega^2 - \omega'^2).R$

Total frictional torque acting $= \mu\, mr(\omega^2 - \omega'^2).R.n$ (8.40)

If p is the maximum pressure intensity exerted on the shoe then

$$mr(\omega^2 - \omega'^2) = p.lb$$

where l and b are the contact length and width of each shoe.

Usually, the clearance between the shoe and the rim is very small and is neglected. However, it can be taken into account if need be.

Example 8.15 ***The inner and the outer radii of a single plate clutch are 40 mm and 80 mm respectively. Determine the maximum, minimum and the average pressure when the axial force is 3 kN.***

Solution: The maximum pressure will be at the inner radius,

$$F = 2\pi p_i R_i (R_o - R_i)$$
$$3000 = 2\pi \times p_i \times 0.04\,(0.08 - 0.04)$$
$$p_i = 298.4 \times 10^3 \text{ N/m}^2 \text{ or } 298.4 \text{ kN/m}^2$$

or maximum pressure = 298.4 kN/m^2

The minimum pressure will be at the outer radius,

$$F = 2\pi p_i R_i (R_o - R_i)$$
$$3000 = 2\pi \times p_i \times 0.08\,(0.08 - 0.04)$$
$$p_i = 149.2 \times 10^3 \text{ N/m}^2 \text{ or } 149.2 \text{ kN/m}^2$$

or minimum pressure = 149.2 kN/m^2

$$\text{The average pressure} = \frac{\text{Total normal force}}{\text{Cross-sectional area}}$$

$$= \frac{3000}{\pi(0.08^2 - 0.04^2)}$$

$= 198.9 \times 10^3$ N/m^2 or 198.9 kN/m^2

Example 8.16 *A single plate clutch is required to transmit 8 kW at 1000 rpm. The axial pressure is limited to 70 kN/m^2. The mean radius of the plate is 4.5 times the radial width of the friction surface. If both the sides of the plate are effective and the coefficient of friction is 0.25, find the*

(i) inner and the outer radii of the plate and the mean radius

(ii) width of the friction lining

Solution:

$P = 8$ kW $\qquad \mu = 0.25$

$N = 1000$ rpm

$$R_m = \frac{R_o + R_i}{2} = 4.5(R_o - R_i)$$

or $\quad R_o + R_i = 9(R_o - R_i)$

or $\quad 8R_o = 10R_i$

$R_o = 1.25 R_i$

In case of power transmission through a clutch, it is safer to use the expressions obtained by uniform wear theory. In that case maximum pressure is at the inner radius, i.e., $p_i = 70$ kN/m^2

$P = T\omega$

or

$$8000 = T \times \frac{2\pi \times 1000}{60}$$

$T = 76.39$ N.m

(i) $$T = \frac{\mu F}{2}(R_o + R_i) \times n$$

(n = number of surfaces)

$$= \frac{\mu}{2}[2\pi p_i R_i (R_o - R_i)](R_o + R_i) \times 2$$

$$= \mu[2\pi \times 70 \times 10^3 \times R_i(1.25R_i - R_i)](1.25R_i + R_i)$$

$$= 0.25 \times 2 \times \pi \times 70 \times 10^3 \times 0.5625R_i^3$$

$76.39 = 61\,850R_i^3$

$\therefore R_i = 0.1073$ m

$R_o = 1.25 \times 0.1073 = 0.1341$ m

$R_m = (R_o - R_i)\,4.5 = (0.1341 - 0.1073) \times 4.5 = 0.1207$ m

(ii) $w = R_m/4.5 = 0.1207/4.5$

$= 0.0268$ m or 26.8 mm

Example 8.17 *A single-plate clutch transmits 25 kW at 900 rpm. The maximum pressure intensity between the plates is 85 kN/m^2. The outer diameter of the plate is 360 mm. Both the sides of the plate are effective and the coefficient of friction is 0.25. Determine the*

(i) inner diameter of the plate

(ii) axial force to engage the clutch

Solution:

$P = 25$ kW $\qquad \mu = 0.25$

$N = 900$ rpm $\qquad R_o = 0.18$ m

$p_i = 85$ kN/m^2

Now, $P = T\omega$

$$25\,000 = T \times \frac{2\pi \times 900}{60}$$

$T = 265.26$ N.m

(i) $$T = \frac{\mu F}{2}(R_o + R_i) \times n$$

(n = number of surfaces)

$$= \frac{\mu}{2}[2\pi p_i R_i (R_o - R_i)](R_o + R_i) \times n$$

$265.26 = 0.25 \times \pi \times 85\,000 \times R_i$
$(0.18 - R_i)(0.18 + R_i) \times 2$

$R_i = [(0.18)^2 - R_i^2] = 0.001987$

or $\quad 0.0324R_i - R_i^3 = 0.001987$

Solving the equation by trial and error method,

$R_i = 0.1315$ m or 131.5 mm

(ii) $F = 2\pi p_i R_i (R_o - R_i) \times n$

$= 2\pi \times 85\,000 \times 0.1315\,(0.18 - 0.1315) \times 2$

$= 6812$ N or 6.812 kN

Example 8.18 *A friction clutch is used to rotate a machine from a shaft rotating at a uniform speed of 250 rpm. The disc-type clutch has both of its sides effective, the coefficient of friction being 0.3. The outer and the inner diameters of the friction plate are 200 mm and 120 mm respectively. Assuming uniform*

wear of the clutch, the intensity of pressure is not to be more than 100 kN/m². If the moment of inertia of the rotating parts of the machine is 6.5 kg.m², determine the time to attain the full speed by the machine and the energy lost in slipping of the clutch.

What will be the intensity of pressure if the condition of uniform pressure of the clutch is considered? Also, determine the ratio of power transmitted with uniform wear to that with uniform pressure.

Solution:

$p_i = 100 \times 10^3$ N/m² $\quad R_o = 0.1$ m

$I = 6.5$ kg.m² $\quad R_i = 0.06$ m

$\mu = 0.3$ $\quad N = 250$ rpm

$n = 2$ (both sides effective)

(a) *With uniform wear*

Force/surface, $F = 2\,\pi\, p_i\, R_i\,(R_o - R_i)$

$= 2\,\pi \times 100 \times 10^3 \times 0.06(0.1 - 0.06) = 1508$ N

$$T = \frac{\mu F}{2}(R_o + R_i) \times n$$

$$= \frac{0.3 \times 1508}{2}(0.1 + 0.06) \times 2$$

$= 72.38$ N.m

$$P = T \times \omega = 72.38 \times \frac{2\pi \times 250}{60} = 1895 \text{ W}$$

Also $T = I\alpha$ $\quad$ (α is angular acceleration)

$72.38 = 6.5 \times \alpha$

or $\alpha = 11.135$ rad/s²

or $\dfrac{\omega}{t} = 11.135$

or $\dfrac{2\pi \times 250}{60 \times t} = 11.135$

or $t = 2.35$ s

Thus, the full speed is attained by the machine in 2.35 seconds.

(b) *During the slipping period*

Angle turned by the driving shaft,

$$\theta_1 = \omega t = \frac{2\pi N}{60} t = \frac{2\pi \times 250}{60} \times 2.35$$

$= 61.5$ rad

Angle turned by the driven shaft,

$$\theta_2 = \omega_o t + \frac{1}{2}\alpha t^2 \qquad (\omega_o = 0)$$

$$= 0 + \frac{1}{2} \times 11.135 \times (2.35)^2$$

$= 30.75$ rad

Energy lost in friction $= T(\theta_1 - \theta_2)$

$= 72.38 \times (61.5 - 30.75)$

$= 2226$ N.m or 2.226 kN.m

(c) *With uniform pressure*

$$P = \frac{F}{\pi(R_o^2 - R_i^2)} = \frac{1508}{\pi[(0.1)^2 - (0.06)^2]}$$

$= 75\,000$ N/m² or 75 kN/m²

$$T = \frac{2}{3}\mu F\left[\frac{R_o^3 - R_i^3}{R_o^2 - R_i^2}\right] \times n$$

$$= \frac{2}{3} \times 0.3 \times 1508\left[\frac{(0.1)^3 - (0.06)^3}{(0.1)^2 - (0.06)^2}\right] \times 2$$

$= 73.89$ N.m

$$P = 73.89 \times \frac{2\pi \times 250}{60} = 1934 \text{ W}$$

$$\frac{\text{Power with uniform wear}}{\text{Power with uniform pressure}} = \frac{1895}{1934} = \underline{0.98}$$

Example 8.19 ***A single-plate clutch used to drive a rotor from the motor shaft has the following data:***

Internal diameter of the plate	*= 200 mm*
External diameter of the plate	*= 240 mm*
Spring force pressing the plates	*= 600 N*
Mass of the rotor	*= 1200 kg*
Radius of gyration of the rotor	*= 200 mm*
Mass of motor armature and shaft	*= 750 kg*
Radius of gyration of motor and shaft	*= 220 mm*
Coefficient of friction	*= 0.32*

Both sides of the plate are effective. The driving motor is brought to a speed of 1260 rpm and then suddenly the current is switched off and the clutch is engaged. Determine the

(i) final speed of the rotor *and the motor and the time to attain this speed*

(ii) kinetic energy lost during slipping

(iii) slipping time, if a constant resisting torque of 64 N.m exists on the armature shaft

(iv) slipping time if instead of a resisting torque, a constant driving torque of 64 N.m exists on the armature shaft

Solution:

$$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 1260}{60} = 132 \text{ rad/s}$$

Moment of inertia of motor, $I_m = 750 \times (0.22)^2$
$= 36.3 \text{ kg.m}^2$

Moment of inertia of rotor, $I_r = 1200 \times (0.2)^2$
$= 48 \text{ kg.m}^2$

Friction torque, $T = \frac{\mu F}{2}(R_o + R_i) \times n$

$= \frac{0.32 \times 600}{2}(0.12 + 0.1) \times 2$

$= 42.24$ N.m

(i) Final speed of motor = Final speed of rotor

or $\omega_m + \alpha_m t = \omega_r + \alpha_r t$

or $\omega_m - \frac{T}{I_m} t = \omega_r + \frac{T}{I_r} t$

or $132 - \frac{42.24}{36.3} t = 0 + \frac{42.24}{48} t$

or $2.0436\, t = 132$

$t = \underline{64.6 \text{ s}}$

Final speed $= \frac{42.24}{48} t = \frac{42.24}{48} \times 64.6$
$= 56.85$ rad/s

$= \frac{60}{2\pi} \times 56.85 = \underline{542.9 \text{ rpm}}$

(ii) Angle turned by the driving shaft,

$\theta_1 = \omega t - \frac{1}{2}\alpha t^2$

$= 132 \times 64.6 - \frac{1}{2}.\frac{42.24}{36.3} \times 64.6^2$

$= 6099$ rad

Angle turned by the rotor, $\theta_2 = \omega t + \frac{1}{2}\alpha t^2$

$= 0 + \frac{1}{2}.\frac{42.24}{48} \times 64.6^2$

$= 1836$ rad

Energy lost in friction $= T(\theta_1 - \theta_2) = 42.24 \times (6099 - 1836) = 180\,060$ N or $\underline{180.06 \text{ kN}}$

(iii) Total resisting torque on the armature
$= -42.24 - 64 = -106.24$ N.m

$\therefore\ 132 - \frac{106.24}{36.3} t = 0 + \frac{42.24}{48} t$

$3.8067\, t = 132$

$t = \underline{34.7 \text{ s}}$

(iv) Net resisting torque on the armature
$= -42.24 + 64 = 21.76$ N.m

i.e. it is accelerating torque.

$\therefore\ 132 + \frac{21.76}{36.3} t = 0 + \frac{42.24}{48} t$

$0.2806\, t = 132$

$t = \underline{470.5 \text{ s}}$

Example 8.20 *An engine is coupled to a rotating drum by a single disc friction clutch having both of its sides lined with friction material. Axial pressure on the disc is 1 kN. Inner and outer diameters of the disc are 280 mm and 360 mm respectively. The engine develops a constant torque of 36 N.m and the inertia of its rotating parts is equivalent to that of a flywheel of 30-kg mass and a radius of gyration of 280 mm. The mass and radius of gyration of the drum are 50 kg and 420 mm respectively and the torque to overcome the friction is 6 N.m. The clutch is engaged when the engine speed is 480 rpm and the drum is stationary. Assuming the coefficient of friction to be 0.3, determine the*

(i) duration of slipping

(ii) speed when the clutch slip ceases

(iii) total time taken for the drum to reach a speed of 480 rpm

Solution:

$F = 1000 \text{ N/m}^2$ $\quad N = 480$ rpm

$R_o = 0.18$ m $\quad R_i = 0.14$ m

$\mu = 0.3$ $\quad n = 2$ (both sides effective)

$$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 480}{60} = 16\pi \text{ rad/s}$$

Moment of inertia of engine, $I_e = 30 \times (0.28)^2$
$= 2.35 \text{ kg.m}^2$

Moment of inertia of drum, $I_d = 50 \times (0.42)^2$
$= 8.82 \text{ kg.m}^2$

Friction torque, $T = \frac{\mu F}{2}(R_o + R_i) \times n$

$$= \frac{0.3 \times 1000}{2}(0.18 + 0.14) \times 2$$

$= 96$ N.m

(i) Net resisting torque on the armature $= -96 + 36 = -60$ N.m

Net accelerating torque on the drum $= 96 - 6 = 90$ N.m

Final speed of engine = Final speed of rotor

or $\quad \omega_e + \alpha_e t = \omega_d + \alpha_d t$

or $\quad \omega_e - \frac{T}{I_e} t = \omega_d + \frac{T}{I_d} t$

$\therefore \quad 16\pi - \frac{60}{2.35} t = 0 + \frac{90}{8.82} t$

$35.714\, t = 16\pi$

$t = \underline{1.407 \text{ s}}$

(ii) The speed when the slip ceases =

$\frac{90}{8.82} \times 1.407 = 14.357$ rad/s

or $\frac{60}{2\pi} \times 14.357 = \underline{137.1 \text{ rpm}}$.

(iii) After the slipping ceases,

Net accelerating torque on the engine and the drum $= 36 - 6 = 30$ N.m

Net moment of inertia of the engine and the drum $= 2.35 + 8.82 = 11.17$ kg.m^2

Now, final speed = initial speed + acceleration × time

$16\pi = 14.357 + \frac{30}{11.17} t$

$2.685\, t = 35.908$

$t = 13.37$ s

$\therefore$ total time taken $= 1.41 + 13.37 = \underline{14.78 \text{ s}}$

Example 8.21 ***If the capacity of a single-plate clutch decreases by 13 % during the initial wear period, determine the minimum value of the ratio of internal diameter to external diameter for the same axial load. Consider both the sides of the clutch plate to be effective.***

Solution: A new clutch has a uniform pressure distribution, but after the initial wear the clutch exhibits the characteristics of uniform wear. Capacity of a clutch means the maximum torque transmitted. Thus, according to the given condition,

$$T_{wear} = 0.87\, T_{pressure}$$

$$\frac{\mu F}{2}(R_o + R_i) \times n = 0.87 \times \frac{2}{3}\mu F\left[\frac{R_o^3 - R_i^3}{R_o^2 - R_i^2}\right] \times n$$

or $\quad (R_o + R_i) = 1.16\left[\frac{R_o^3 - R_i^3}{R_o^2 - R_i^2}\right]$

$$(R_o + R_i)(R_o^2 - R_i^2) = 1.16\,(R_o^3 - R_i^3)$$

$$(R_o + R_i)(R_o + R_i)(R_o - R_i) = 1.16\,(R_o - R_i)\,(R_o^2 + R_oR_i + R_i^2)$$

$$(R_o + R_i)^2 = 1.16\,(R_o^2 + R_oR_i + R_i^2)$$

$$(R_o^2 + 2R_oR_i + R_i^2) = 1.16\,(R_o^2 + R_oR_i + R_i^2)$$

$$0.16\,R_o^2 + 0.16R_i^2 - 0.84\,R_o R_i = 0$$

Dividing throughout by $0.16\, R_o^2$,

$$1 + \left(\frac{R_i}{R_o}\right)^2 - 5.25\left(\frac{R_i}{R_o}\right) = 0$$

or $\quad \left(\frac{R_i}{R_o}\right)^2 - 5.25\left(\frac{R_i}{R_o}\right) + 1 = 0$

Taking $\frac{R_i}{R_o} = r$

$$r^2 - 5.25\,r + 1 = 0$$

or $\quad r = \frac{5.25 \pm \sqrt{5.25^2 - 4}}{2}$

$$= \frac{5.25 \pm 4.854}{2}$$

Positive value is not possible as ratio r cannot be more than 1.

$\therefore \quad r = \frac{5.25 - 4.854}{2} = 0.198$

or $\quad \frac{R_i}{R_o} = \underline{0.198}$

Example 8.22 *A multi-plate disc clutch transmits 55 kW of power at 1800 rpm. Coefficient of friction for the friction surfaces is 0.1. Axial intensity of pressure is not to exceed 160 kN/m². The internal radius is 80 mm and is 0.7 times the external radius. Find the number of plates needed to transmit the required torque.*

Solution:

$p_i = 160 \times 10^3$ N/m² $\qquad R_i = 0.08$ m

$\mu = 0.1 \qquad R_o = \dfrac{0.08}{0.7} = 0.1143$ m

$N = 1800$ rpm

$P = 55$ kW

Assuming uniform wear conditions,

$$F = 2\pi p_i r_i (R_o - R_i)$$
$$= 2\pi \times 160 \times 10^3 \times 0.08\,(0.1143 - 0.08)$$
$$= 2759 \text{ N}$$

$$T = \frac{1}{2}\mu F(R_o + R_i)$$
$$= \frac{1}{2} \times 0.1 \times 2759 \times (0.1143 + 0.08)$$
$$= 26.78 \text{ N.m/surface}$$

Total torque transmitted

$$= \frac{P}{\omega} = \frac{55000}{\dfrac{2\pi \times 1800}{60}} = 291.8 \text{ N.m}$$

Number of friction surfaces required $= \dfrac{291.8}{26.78}$

$= 10.9$ or 11 surfaces

In all, there will be 12 plates. 6 plates (rings) revolve with the driving or engine shaft and the other 6 with the driven shaft.

Example 8.23 *A multi-plate disc clutch transmits 30 kW of power at 1800 rpm. It has four discs on the driving shaft and three discs on the driven shaft providing six pairs of contact surfaces. The external and internal diameters of the contact surfaces are 200 mm and 100 mm respectively. Assuming the clutch to be new, find the total spring load pressing the plates together. Coefficient of friction is 0.3.*
Also, determine the maximum power transmitted when the contact surfaces have worn away by 0.4 mm. There are 8 springs and the stiffness of each spring is 15 kN/m.

Solution

$$\omega = \frac{2\pi \times 1800}{60} = 60\pi \text{ rad/s}$$

A new clutch has a uniform pressure distribution, but after the initial wear the clutch exhibits the characteristics of uniform wear.

$P = T\omega$ or $30\,000 = T \times 60\pi$ or $T = 159.15$ N.m

Thus, torque transmitted by new clutch

$$= \frac{2}{3}\mu F\left[\frac{R_o^3 - R_i^3}{R_o^2 - R_i^2}\right] \times n$$

or $\qquad 159.15 = \dfrac{2}{3} \times 0.3F\left[\dfrac{100^3 - 50^3}{100^2 - 50^2}\right] \times 6$

or $\qquad F = 1137$ N

When the surfaces are worn out

Contact surfaces = number of pairs of contact × 2 = 6 × 2 = 12

Total wear = number of surfaces × wear of each surface = 12 × 0.4 = 4.8 mm

Stiffness of each spring = 15 kN/m = 15 N/mm

Total stiffness of springs = stiffness × number of springs = 15 × 8 = 120 N/mm

∴ reduction in spring force = 120 × 4.8 = 576 N

New axial load = 1137 – 576 = 561 N

$$T = \frac{1}{2}\mu F(R_o + R_i).n = \frac{1}{2} \times 0.3 \times 561 \times$$
$$(100 + 50) \times 6 = 75\,735 \text{ N.mm} = 75.735 \text{ N.m}$$

Maximum power transmitted = $75.735 \times 60\pi$

= 14 276 W = 14.276 kW

Example 8.24 *A torque of 350 N.m is transmitted through a cone clutch having a mean diameter of 300 mm and a semi-cone angle of 15°. The maximum normal pressure at the mean radius is 150 kN/m². The coefficient of friction is 0.3.*

Calculate the width of the contact surface. Also, find the axial force to engage the clutch.

Solution:

$T = 350$ N.m $\qquad R_m = 0.15$ m

$P_m = 150$ kN/m^2 $\qquad \mu = 0.3$

$\alpha = 15°$

$$T = \mu F_n R_m$$

$$350 = 0.3 \times F_n \times 0.15$$

$$F_n = 7\,778 \text{ N}$$

and

$$F_n = 2\pi p_m R_m b$$

$$7\,778 = 2\pi \times 150 \times 10^3 \times 0.15 \times b$$

$b = 0.055$ m or <u>55 mm</u>

Axial force, $F = F_n \sin\alpha = 7\,778 \sin 15°$

$= $ <u>2012.4 N</u>

Example 8.25 ***The semi-cone angle of a cone clutch is 12.5° and the contact surfaces have a mean diameter of 80 mm. Coefficient of friction is 0.32. What is the minimum torque required to produce slipping of the clutch for an axial force of 200 N?***

If the clutch is used to connect an electric motor with a stationary flywheel, determine the time needed to attain the full speed and the energy lost during slipping. Motor speed is 900 rpm and the moment of inertia of the flywheel is 0.4 kg.m².

Solution:

$R_m = 0.04$ m $\qquad \mu = 0.32$

$F = 200$ N $\qquad \alpha = 12.5°$

$N = 900$ rpm $\qquad I = 0.4$ kg.m^2

$T = I\,\alpha_a \qquad$ (α_a = angular acceleration)

or $\quad \mu \dfrac{F}{\sin\alpha} R_m = I.\alpha_a$

or $\quad 0.32 \times \dfrac{200}{\sin 12.5°} \times 0.04 = 0.4\alpha_a$

$\alpha_a = 29.57$ rad/s^2

and $\quad T = 0.4 \times 29.57$

$= 11.828$ N.m

or $\quad \dfrac{\omega}{t} = 29.57$

or $\quad \dfrac{2\pi \times 900}{60t} = 29.57$

$t = $ <u>3.187 s</u>

During slipping period

Angle turned by driving shaft,

$$\theta_1 = \omega t = \frac{2\pi \times 900}{60} \times 3.187 = 300.4 \text{ rad}$$

Angle turned by driven shaft, $\theta_2 = \omega_o t + \dfrac{1}{2}\alpha_a t^2$

$$= 0 + \frac{1}{2} \times 29.57 \times (3.187)^2 = 150.2 \text{ rad}$$

Energy lost in friction $= T(\theta_1 - \theta_2)$

$= 11.828\,(300.4 - 150.2) = $ <u>1776.5 N.m</u>

Alternatively,

energy supplied $= T\omega \times$ time

$$= 11.828 \times \frac{2\pi \times 900}{60} \times 3.187 = 3553 \text{ N.m}$$

Energy of flywheel

$$= \frac{1}{2} I\omega^2 = \frac{1}{2} \times 0.4 \times \left(\frac{2\pi \times 900}{60}\right)^2 = 1776.5 \text{ N.m}$$

Energy lost $= 3553 - 1776.5 = 1776.5$ N.m

Example 8.26 ***A cone clutch with a semi-cone angle of 15° transmits 10 kW at 600 rpm. The normal pressure intensity between the surfaces in contact is not to exceed 100 kN/m². The width of the friction surfaces is half of the mean diameter. Assume μ = 0.25. Determine the***

(i) outer and inner diameters of the plate

(ii) width of the cone face

(iii) axial force to engage the clutch.

Solution:

$P = 10$ kW $\qquad p = 100$ kN/m^2

$N = 600$ rpm $\qquad \mu = 0.25$

$\alpha = 15° \qquad b = \dfrac{d_m}{2} = R_m$

(i) $b = R_m$

or

$$\frac{R_o - R_i}{\sin\alpha} = \frac{R_o + R_i}{2} \qquad \text{[refer Fig. 8.16]}$$

or

$$R_o - R_i = \frac{\sin 15°}{2}(R_o + R_i)$$

$$= 0.129R_o + 0.129R_i$$

or $R_o = 1.296\, R_i$

Now,

$$P = T.\,\omega$$

$$10\,000 = T.\,\frac{2\pi \times 600}{60}$$

$$T = 159 \text{ N.m}$$

Intensity of pressure is maximum at the inner radius (uniform wear theory).

$$T = \frac{\mu F_n}{2}(R_o + R_i)$$

$$= \frac{\mu F}{2\sin\alpha}(R_o + R_i)$$

$$= \frac{\mu}{2\sin\alpha}[2\pi p_i R_i (R_o - R_i)](R_o + R_i)$$

$$= \frac{\mu\pi p_i}{\sin\alpha} R_i (R_o^2 - R_i^2)$$

$$159 = \frac{0.25\pi \times 100 \times 10^3}{\sin 15°} R_i[(1.296R_i)^2 - R_i^2]$$

$$= \frac{0.25\pi \times 100 \times 10^3}{\sin 15°} \times 0.6796 R_i^3$$

$R_i = 0.0917$ m or <u>91.7mm</u>

$R_o = 91.7 \times 1.296 =$ <u>118.8 mm</u>

(ii) $b = \frac{R_o - R_i}{\sin\alpha} = \frac{118.8 - 91.7}{\sin 15°} =$ <u>105 mm</u>

(iii) Axial force, $F = 2\,\pi p_i R_i\,(R_o - R_i)$

$= 2\pi \times 100 \times 10^3 \times 0.0917\,(0.1188 - 0.0917)$

= <u>1561 N</u>

Example 8.27 *A centrifugal clutch transmits 20 kW of power at 750 rpm. The engagement of the clutch commences at 70 per cent of the running speed. The inside diameter of the drum is 200 mm and the distance of the centre of mass of each shoe is 40 mm from the contact surface. Determine the*

(i) mass of each shoe

(ii) net force exerted by each shoe on the drum surface

(iii) power transmitted when the shoe is worn 2 mm and is not readjusted

Assume μ to be 0.25 and stiffness of the spring 150 kN/m.

Solution:

$P = 20$ kW $R = 0.2$ m

$N = 750$ rpm $r = 0.2 - 0.04 = 0.16$ mm

$\mu = 0.25$

(i) $\omega = \frac{2\pi \times 750}{60} = 78.5$ rad/s

$\omega' = 0.7 \times 78.5 = 55$ rad/s

$P = T\,\omega$

$20\,000 = T \times 78.5$

T = 254.8 N.m

Total frictional torque acting

$= \mu\, mr\,(\omega^2 - \omega'^2).R.n$

$254.8 = 0.25 \times m \times 0.16(78.5^2 - 55^2) \times 0.2 \times 4$

$= 100.4 \times m$

$m =$ <u>2.538 kg</u>

(ii) Net force exerted by each shoe on the drum surface

$= mr(\omega^2 - \omega'^2)$

$= 2.538 \times 0.16\,(78.5^2 - 55^2)$

= <u>1274 N</u>

(iii) Spring force exerted by each spring

$= 2.538 \times 0.16 \times 55^2 = 1228.4$ N

When the shoe wears 2 mm, each shoe has to move forward by 2 mm. This increases the distance of the centre of mass of the shoe from 160 mm to 162 mm. Also, the spring force is increased due to its additional displacement of 2 mm.

Additional spring force = Stiffness × Displacement

$= 150 \times 10^3 \times 0.002$

= 300 N

Total spring force = 1228.4 + 300 = 1528.4 N

Net force exerted by each shoe on the drum surface

$= mr\omega^2 - 1528.4$

$= 2.538 \times 0.162 \times 78.5^2 - 1528.4$

= <u>1005.2 N</u>

Total frictional torque acting = $\mu F.R.n$

$T = 0.25 \times 1005.2 \times 0.2 \times 4$

= 201.04 N.m

$P = T.\,\omega$

$= 201.04 \times 78.5$

= 15 782 W or <u>15.782 kW</u>

8.10 ROLLING FRICTION

When a ball rolls over a flat surface, the contact is theoretically at a point. Similarly, when a cylinder rolls over the surface, the contact is along a line parallel to the axis. However, the ball or the cylinder possesses weight and due to the pressure of the same, deformation of the flat surface or of the rolling body or of both takes place. The amount of deformation will depend upon the elasticity of the materials in contact and the pressure. With harder surfaces, the deformation is extremely small. The deformation causes the two surfaces to have area of contact rather than point or line contact.

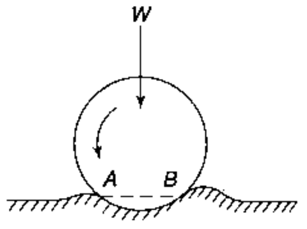


Fig. 8.18

Figure 8.18 shows a ball rolling on a flat surface. The ball is assumed to be made of a very hard material so that its deformation is negligible. The material of the dented surface is stretched at the place of contact as the curved length AB is greater than the flat length AB.

Now, if the ball is to roll to the right, the material of the flat surface must be stretched to accommodate the curved surface of the ball. The stretching material slides against the surface of the ball which causes frictional resistance. The frictional resistance also occurs when the material left behind again contracts and slides against the surface of the ball. This friction is known as *rolling friction*.

8.11 ANTI-FRICTION BEARINGS

Ordinarily, if a shaft revolves in a bearing, it has a sliding motion. But if balls or rollers are used between the shaft and the journal as shown in Figs 8.19 and 8.20 then rolling occurs between the journal and the rollers as well as between the rollers and the shaft. Usually, the balls or rollers are made of hardened materials such as chromium steel or chrome–nickel steel.

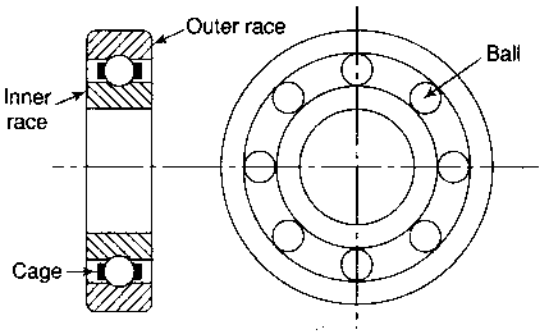


Fig. 8.19

1. Ball Bearings

A ball bearing consists of a number of hardened balls mounted between two hardened races. The inner race is fitted to the shaft and the outer race is a tight fit into the bearing housing. Thus, there is no relative movement between the shaft and the inner race, and the outer race and the housing. There are shallow grooves in the races having a slight larger radius than that of the balls to accommodate the balls. A light brass cage keeps the balls at a fixed distance from one another (Fig. 8.19).

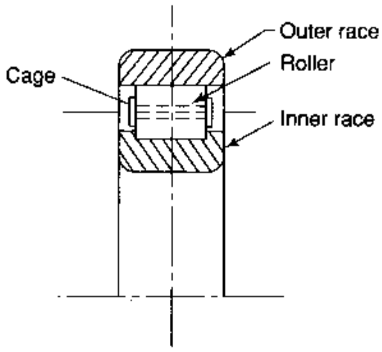


Fig. 8.20

Since the balls as well as the races are very hard, distortion of each is little and the rolling friction is very low.

The friction of ball bearings is slightly higher when lubricated than when dry. However, a small amount of lubricant is useful to prevent rust formation.

2. Roller Bearings

These are similar to ball bearings where the balls are replaced by hardened cylindrical rollers. These bearings can carry heavier loads.

Figure 8.20 shows a radial roller bearing. This type of bearing will carry only the radial loads or loads perpendicular to the shaft axis. The outer race is plain whereas the inner race has a groove to accommodate the rollers. A cage keeps the rollers at a uniform distance apart.

When the rollers are small and used without a cage, they are known as *needles.* Such bearings are used to carry heavy loads.

Roller bearings are not used for misaligned shafts.

8.12 GREASY FRICTION

If two metallic surfaces are wetted with a small amount of lubricant, a very thin film of the same is formed on each of the surfaces. This thin film is of molecular thickness. It adheres to the surface and is known as *adsorbed film.* It has been found that when the two surfaces are placed in contact, the coefficient of friction between them is considerably reduced compared with when the surfaces are dry and unlubricated.

The property of a lubricant to form a layer of molecular thickness (adsorbed film) on a metallic surface is known as its *oiliness.* If two lubricants of equal viscosity are used to lubricate two metallic surfaces under identical conditions, it is found that one reduces the friction more than the other and is said to have greater oiliness.

The friction of two surfaces, when they are wetted with an extreme thin layer of lubricant and metal-to-metal contact can take place between high spots, is known as *greasy friction* or *boundary friction.*

The laws governing greasy friction are similar to those for solid or dry friction.

8.13 GREASY FRICTION AT A JOURNAL

Greasy or boundary friction occurs in heavily loaded, slow-running bearings. In this type of friction, the frictional force is assumed to be proportional to the normal reaction.

When a shaft rests in the bearing [Fig. 8.21(a)], its weight **W** acts through its centre of gravity. The reaction of the bearing acts in line with **W** in the vertically upward direction. The shaft rests at the bottom of the bearing at A and metal to metal contact exists between the two.

When a torque is applied to the shaft, it rotates and the seat of pressure creeps or climbs up the bearing in a direction opposite to that of rotation. Metal-to-metal condition still exists and greasy friction criterion applies as the oil film will be of molecular thickness. The common normal at B between the two surfaces in contact passes through the centre of the shaft.

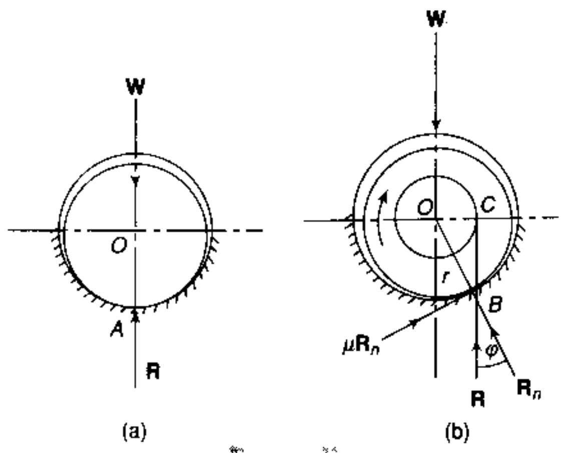


Fig. 8.21

Let $\mathbf{R}_n$ = normal (radial) reaction at B

$\mu\mathbf{R}_n$ = friction force, tangential to the shaft

The normal reaction and the friction force can be combined into a resultant reaction **R** inclined at an angle φ to $\mathbf{R}_n$ [Fig. 8.21(b)].

Now the shaft is in equilibrium under the following forces:

(i) weight **W**, acting vertically downwards, and

(ii) reaction **R**.

For equilibrium, **R** must act vertically upwards and must be equal to **W**. However, the two forces **W** and **R** will be parallel and constitute a couple.

Let OC = Perpendicular to R from O

$$\text{Friction couple (torque)} = W \times OC = Wr \sin \varphi \qquad \left(\because \sin \varphi = \frac{OC}{r}\right)$$

$$\approx Wr \tan \varphi \qquad \text{(as } \varphi \text{ is small)}$$

$$\approx Wr\,\mu \qquad (\mu = \tan \varphi)$$

This couple must be equal and opposite to the couple or torque producing motion.

A circle drawn with OC (or $r \sin\varphi \approx r \tan \varphi \approx r\mu$) as radius is known as the *friction circle* of the journal.

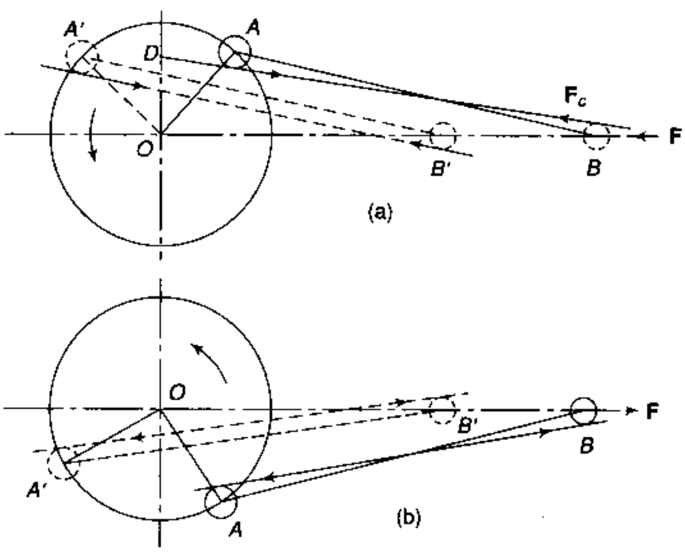
Thus, the effect of friction is equivalent to displacing the reaction through a distance equal to $r \sin \varphi$ or such that it is tangential to the friction circle.

8.14 FRICTION AXIS OF A LINK

In a pin-jointed mechanism, usually, it is assumed that the resulting thrusts (axial forces) in links act along the longitudinal axes of the links. But as the laws of dry friction are similar to those of greasy friction, the friction at pin-joint acts in the same way as that for a journal (rotating shaft) revolving in a bearing. In a journal bearing, the resultant force on a journal is tangential to the friction circle. Similarly, in pin-jointed links, the line of thrust on a link is tangential to the friction circles at the pin-joints. The net effect of all this is to shift the axis along which the thrust acts. The new axis is knows as the *friction axis* of the link.

1. Slider-crank Mechanism

Figure 8.22(a) shows a slider-crank mechanism in which **F** is the thrust on the slider. If the effect of friction is neglected, the force **F** will induce a thrust $\mathbf{F}_c$ in the connecting rod along its axis. However, due to friction, the friction axis will be along a tangent to the friction circles at the joints A and B.



Let r_a = radius of pin at A

r_b = radius of pin at B

μ_a = coefficient of friction at A

μ_b = coefficient of friction at B

Therefore, the radius of friction circle at $A = \mu_a r_a$

The radius of friction circle at $B = \mu_b r_b$

(exactly the two radii will be $r_a \sin \varphi_a$ and $r_b \sin \varphi_b$)

Now, there are four possible ways of drawing a tangent to these circles. To decide about the right one, remember that a friction couple is opposite to the couple or torque producing the motion of a link. Thus, while drawing a tangent to a friction circle, see that the friction couple or torque so obtained is opposite to the direction of rotation of the link. Thus, the position of right tangent depends upon the

1. directions of the external forces on the link, and
2. direction of the motion of the link relative to the link to which it is connected.

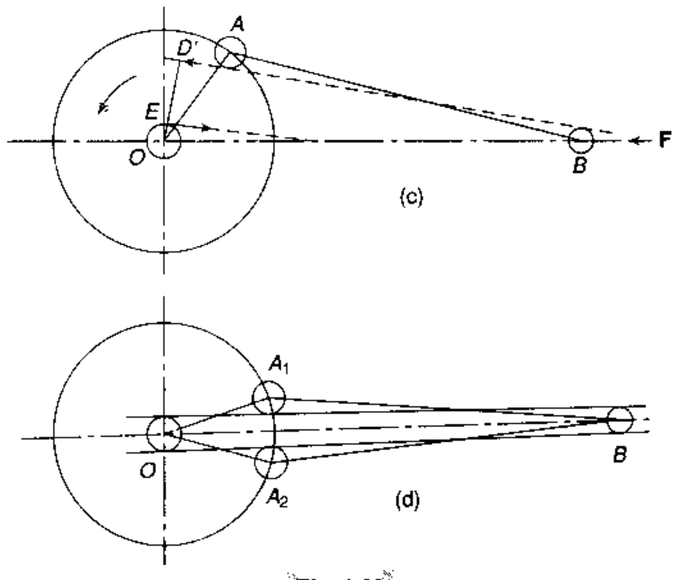


Fig. 8.22

Owing to the force **F** on the piston, compressive forces are developed in the connecting rod and the crank rotates in the counter-clockwise direction. At the end B, the angle OBA is increasing and Thus, the rod rotates clockwise relative to the piston. Therefore, the tangent at B must be on the upper side of the friction circle to give a counter-clockwise friction couple. At the end A, the angle OAB is decreasing and thus, the rod rotates clockwise relative to the crank. Therefore, the tangent at A must be on the lower side of the friction circle to give a counter-clockwise friction couple. The friction axis will be the common tangent to the two friction circles, on the upper side of the circle at B and on the lower side of the circle at A.

Figure 8.22(a) also shows one more position of the crank in the dotted lines. The direction of **F** as well as that of the thrust in the connecting rod is the same as before. However, now the angle $OB'A'$ at B' is decreasing. thus, the rod rotates in the counter-clockwise direction relative to the piston and the tangent is on the lower side of the friction circle to give a clockwise friction couple. The tangent at end A' will be as before.

For the positions of the crank shown in Fig. 8.22(b), the direction of **F** has changed and the rod becomes in tension. The angle $OB'A'$ at B' increases and thus, the rod rotates in the counter clockwise direction. The tangent, therefore, is on the upper side of the circle. At A', the angle $OA'B'$ also increases and the rod rotates clockwise relative to the crank. The tangent is on the upper side of the circle to give a counter-clockwise friction couple. In the same way, the tangent can be drawn for the position OA of the crank.

In case a torque **T** turns the crank counter-clockwise and a load **F** is applied to the piston, then the rod will be in tension instead of compression for the positions of Fig. 8.22(a). For the first position of the crank, the tangent at A will be on the upper side and at B on the lower side.

Another method to know the friction axis out of the four possible tangents is to select the one that gives the least intercept from O on a vertical through O such as OD if **F** is the driving force. In case **T** is the driving torque at the crank, the intercept would be maximum.

Reaction at Crankshaft Bearing (Fig. 8.22c). The crank OA is acted upon by two equal and opposite forces at its two ends O and A forming a couple and a torque. A force equal and opposite to that in the connecting rod acts at the crank end A, the opposite of which acts at end O. As the crankshaft rotates in the counter-clockwise direction, the tangent is on the upper side of the friction circle to give a clockwise friction couple at end O.

The turning moment of the crankshaft = $F_c \times D'E$
where $D'E$ is drawn perpendicular to the friction axis.

Dead-angle and Dead-centre Positions During rotation, when the crank is near the inner dead centre, a position is obtained where the reaction at the crankshaft bearing and the friction axis of the connecting rod become in the same straight line [Fig. 8.22(d)]. These positions of the crank have been shown as OA_1 and OA_2 and are known as *dead-centre positions*. In these positions, the length $D'E$ is zero and, therefore, the turning moment or the torque transmitted is zero.

The angle A_1OA_2 is known as the *dead angle*. A corresponding dead angle at the outer dead-centre position of the crank can also be found in the same way.

2. Four-bar Mechanism

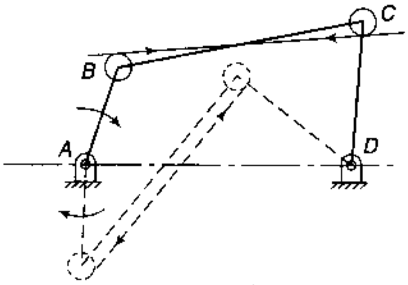


Fig. 8.23

Consider a four-bar mechanism shown in Fig. 8.23. Let the rotation of the driving link AB be clockwise. In the absence of friction, the driving torque induces compressive axial force in the coupler along its axis BC. When friction is considered, it is tangential to the friction circles at B and C.

The angle ABC at B is increasing. Thus, the coupler rotates in the counter-clockwise direction relative to AB. Therefore, the tangent at B is on the upper side of the circle to give a clockwise friction couple. Similarly at C, the angle BCD is decreasing. Thus, BC rotates in the counter-clockwise direction relative to CD. The tangent at C will be on the lower side of the circle so that the resulting friction couple is clockwise. Similarly, the friction axis for the links AB and DC can be drawn.

Example 8.28 ***The length of crank of a slider-crank mechanism is 300 mm and of the connecting rod is 1.25 m. The diameters of the journals at the crosshead, crankpin and the crankshaft are 80 mm, 120 mm and 140 mm respectively. The steam pressure on the piston is 450 kN/m² which has a diameter of 250 mm. Coefficient of friction between the crosshead and the guides is 0.07 and for journals, it is 0.05.***

Find the reduction in the turning moment available at the crankshaft due to friction of the crosshead guides and the journals when the crank has rotated 50° from the inner-dead centre.

Solution: Refer to Fig. 8.24(a).

$AB = 1.25$ m $\qquad \theta = 50°$

$OA = 0.3$ m $\qquad p = 450$ kN/m²

$d = 250$ mm

Neglecting Friction

$$T = F_c \times OC = F_c \times OA \sin(\theta + \beta)$$

Piston at B is in equilibrium under the forces, **F**, $\mathbf{F}_c$ and **R** (reaction of guides).

$$\therefore\ F_c \cos\beta = F \quad \text{or} \quad F_c = \frac{F}{\cos\beta} = \frac{\frac{\pi}{4}d^2 \times p}{\cos\beta}$$

$$\text{Thus } T = \left(\frac{\pi}{4}d^2 p\right) \times OA \frac{\sin(\theta+\beta)}{\cos\beta}$$

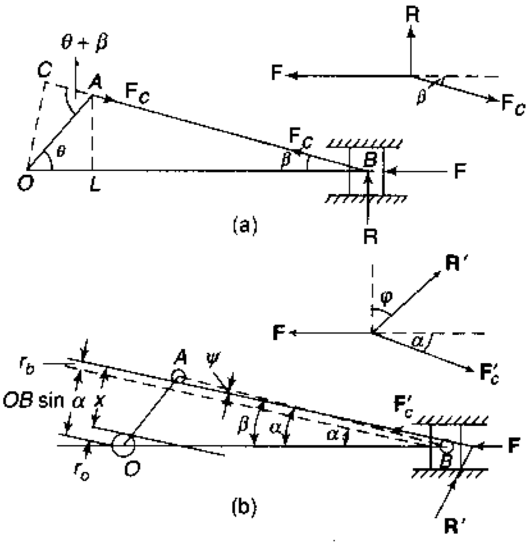


Fig. 8.24

β is given by

$$\beta = \sin^{-}\left(\frac{AL}{AB}\right) = \sin^{-}\left(\frac{OA\sin\theta}{AB}\right)$$

$$= \sin^{-}\left(\frac{0.3\sin 50^\circ}{1.25}\right)$$

$$= \sin^{-}(0.1839) = 10.6^\circ$$

$$T=\left[\frac{\pi}{4}(0.25)^2\times 450\times 10^3\right]\times 0.3\times\frac{\sin(50^\circ+10.6^\circ)}{\cos 10.6^\circ}$$

$$= 22089\times 0.3\times\frac{\sin 60.6^\circ}{\cos 10.6^\circ} = 5873.5 \text{ N.m}$$

Friction Considered Refer to Fig. 8.24(b).

Radius of friction circle at O (crankshaft),

$$r_o = \mu r = 0.05\times\frac{0.14}{2} = 0.0035 \text{ m}$$

Radius of friction circle at A (crankpin),

$$r_a = 0.05\times\frac{0.12}{2} = 0.003 \text{ m}$$

Radius of friction circle at B (crosshead),

$$r_b = 0.05\times\frac{0.08}{2} = 0.002 \text{ m}$$

$$OB = OA\cos\theta + AB\cos\beta$$

$$= 0.3\cos 50^\circ + 1.25\cos 10.6^\circ = 1.4215 \text{ m}$$

Inclination of the friction axis with OB,

$$\alpha = \beta - \psi = \beta - \sin^{-}\left(\frac{r_a + r_b}{AB}\right)$$

$$= 10.6^\circ - \sin^{-}\left(\frac{0.003+0.002}{1.25}\right)$$

$$= 10.6^\circ - 0.23^\circ$$

$$= 10.37^\circ$$

The piston at B is in equilibrium under the action of forces **F**, $\mathbf{F'_c}$ and **R′**.

$$\varphi = \tan^{-1} 0.07 = 4^\circ$$

$$\frac{F'_c}{\sin(90^\circ+\varphi)} = \frac{F}{\sin(90^\circ+\alpha-\varphi)}$$

or $$F'_c = \frac{22\,089\sin(90^\circ+4^\circ)}{\sin(90^\circ+10.37^\circ-4^\circ)} = 22\,172 \text{ N}$$

$$x = OB\sin\alpha + r_b - r_o$$

$$= 1.4215\sin 10.37^\circ + 0.002 - 0.0035$$

$$= 0.2544 \text{ m}$$

$$T' = F'_c\times x = 22\,172\times 0.2544 = 5640.6 \text{ N.m}$$

Reduction in the torque available $= T - T'$

$= 5\,873.5 - 5\,640.6 =$ <u>232.9 N.m</u>

8.15 FILM FRICTION

In case of boundary or greasy friction, a very thin layer of molecular dimensions covers the two surfaces. The friction between the two surfaces acts in the same way as for dry surfaces and the laws are similar except that the friction force is greatly reduced in magnitude.

The friction resistance of two metallic surfaces can be reduced still further if a film of sufficient thickness is introduced in between the two to separate them completely. Now, an extremely thin layer adheres to each of the surfaces and moves with that. Thus, the motion will be due to shearing of the layers of the lubricant rather than due to contact of metallic surfaces. As the sliding or shearing of fluids depends upon the viscosity of the fluid, the friction force will also be depending upon that.

If it is considered that out of the two surfaces, one is at rest then the film of lubricant in contact with the stationary surface will remain at rest and that in contact with the moving surface will move with it. The different layers between the two may be considered to be moving with a speed proportional to its distance from the surface at rest. Thus, the force of friction would be the force necessary to slide these adjacent layers over one another.

For a journal rotating in a bearing under the film lubrication conditions, the frictional resistance is found to be

(a) proportional to the area
(b) proportional to the viscosity of the lubricant
(c) proportional to the speed
(d) independent of the pressure
(e) independent of the materials of the journal and the bearing

The first experiments on film lubrication were carried by Beauchamp Tower in 1883. However, it was Osborne Reynolds who demonstrated that to maintain a film between two surfaces, it is necessary that they are slightly inclined to each other such that a wedge of lubricant is formed between them.

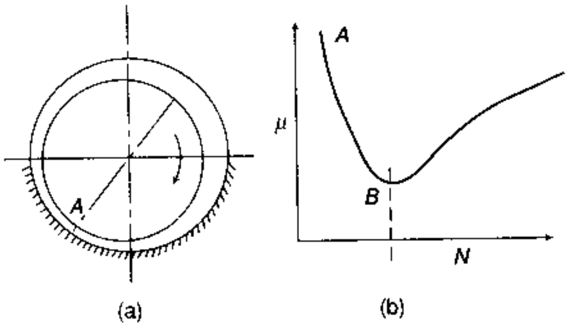


Fig. 8.25

Under greasy friction, in slow running and heavily loaded shafts, the seat of pressure climbs up the bearing in a direction opposite to that of rotation of the shaft and the condition is maintained. However, if the speed is further increased, a film of oil adheres to the surface of the journal which also rotates and becomes of sufficient thickness to lift the journal. The pressure of the oil beneath the journal also rises to support the load.

It is observed that under stable conditions, the journal takes up a new position as shown in Fig. 8.25(a). Two wedges of oil are formed on each side of A, the point of minimum thickness of oil. The point A is also referred as the *point of nearest approach*. The pressure of the oil in the wedge varies from zero at the point of entrance of the oil to a maximum at a point somewhere near A. On the down side of A, the pressure falls to zero before reaching the point of free discharge.

The pressure built up in the oil, due to decrease in area as it approaches the point A, is enough to bear the load of the bearing and separate the two metallic surfaces. The laws of viscous friction apply in this condition.

A plot of graph between the coefficient of friction and the speed [Fig. 8.25(b)] shows that at low speeds the coefficient of friction falls rapidly as the speed increases. This is because more oil is fed between the surfaces lowering the coefficient of friction. From A to B, the conditions vary from greasy friction at A to film friction at B where a complete oil film is framed. As the speed is further increased, the resistance of the oil film increases, increasing the coefficient of friction.

For clockwise rotation of the shaft, the position of its axis shifts towards right in the range of greasy friction and towards left in that of film friction [Figs 8.21(a) and 8.25(a)].

The above discussion may lead to the conclusion that it is possible to have no wear of the two metallic surfaces if film lubrication is adopted. However, in practice, some wear does take place even under the most favourable conditions. The most apparent cause is the failure of the oil film during the starting and stopping of the shaft, when the conditions resemble that of greasy friction, and metal-to-metal contact exists. Metal particles of extremely small size are produced through wear which then float in the lubricant. With time, the number of these particles increases which tends to cause further wear. Moreover, the small particles of solid matter tend to destroy the formation of the viscous oil film. Thus, in actual practice, the working conditions are of a combination of film and greasy lubrication. However, by fitting an oil filter to remove the small particles of solid matter may increase the life of a bearing. Otherwise, the abrasive action will increase the wear, increasing the clearances resulting in insufficient pressure rise to lift the bearing surfaces apart. Thus, the bearing may fail eventually.

8.16 MITCHELL THRUST BEARING

In the previous article, it has been stated that to maintain a film of lubricant between two surfaces, they should be slightly inclined to each other. In a journal bearing, this is achieved by allowing a slight difference in diameters of the journal and the bearing. For flat surfaces, the same purpose is served if one surface has some degree of freedom so that it may become slightly inclined [Fig. 8.26(a)].

To bear axial forces (thrusts) in shafts, usually, a number of collars, working under dry friction conditions have to be used (Sec 8.7). In a Mitchell thrust bearing, the condition of film friction, instead of dry friction, is achieved and only one collar is used which considerably reduces the length of bearing by increasing the allowable pressure. Thus, this type of bearing is useful in transmitting very heavy thrusts, e.g., the thrust of a ship's propeller to the hull.

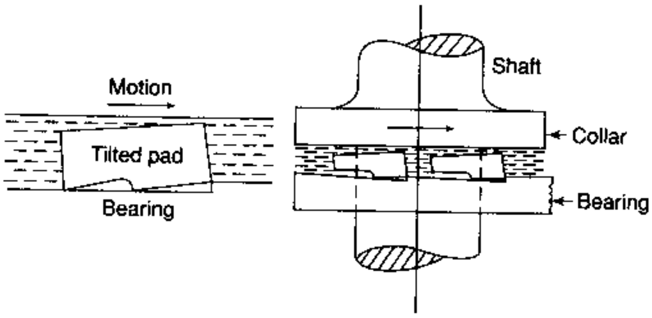


Fig. 8.26

A Mitchell thrust bearing consists of a series of metallic pads arranged around a rotating collar fixed to the shaft [Fig. 8.26(b)]. Each pad is held by the housing of the bearing. Thus, the pad is prevented from rotation but is able to tilt about its stepped edge. When the thrust is transmitted to the pads they can tilt slightly on the edges. The oil carried by the moving collar is dragged around the pad. Thus, an oil film of wedge shape is formed and a considerable pressure is built up to carry the axial load.

The number of pads can be from four to six depending upon the total thrust.

Summary

1. When a body slides over another, the motion is resisted by a force called the *force of friction*. The force arises from the fact that the surfaces, though planed and made smooth, have ridges and depressions that interlock and the relative movement is resisted.
2. In case of lathe slides, journal bearings, etc., where the power transmitted is reduced due to friction, the friction has to be decreased by the use of lubricated surfaces.
3. In processes, where the power is transmitted through friction, attempts are made to increase it to transmit more power. Examples are friction clutches, belt drives, tightening of a nut and bolt, etc.
4. Dry friction is said to occur when there is relative motion between two completely unlubricated surfaces. It is further divided into two types: *solid friction* and *rolling friction*.
5. When the two surfaces in contact have a minute thin layer of lubricant between them, it is known as *skin/ greasy/ boundary friction*.
6. When the two surfaces in contact are completely separated by a lubricant, friction occurs due to the shearing of different layers of the lubricant. This is known as *film friction* or *viscous* friction.
7. The force of solid friction is directly proportional to normal reaction between the two surfaces, opposes the motion, depends upon the materials of the two surfaces and is independent of the area of contact and the velocity of sliding.
8. The maximum value of the angle of inclination of a plane with the horizontal when the body starts sliding of its own is known as the *angle of repose* or *limiting angle of friction*.
9. When a body slides up the plane,
$$\eta = \frac{\cot(\alpha+\theta) - \cot\theta}{\cot\alpha - \cot\theta}$$
If the direction of the applied force is horizontal,
$$\eta = \frac{\tan\alpha}{\tan(\alpha+\varphi)}$$
10. When the body moves down the plane,
$$\eta = \frac{\cot\alpha - \cot\theta}{\cot(\varphi-\alpha) + \cot\theta}$$
If the direction of the applied force is horizontal,
$$\eta = \frac{\tan(\varphi-\alpha)}{\tan\alpha}$$

11. The reversal of the nut is avoided if the efficiency of the thread is less than 50%
12. A wedge is used to raise loads like a screw jack.
13. Efficiency of a wedge
$$= \frac{\cos\varphi' \tan\alpha}{\sin(\alpha + 2\varphi')} \times \frac{\cos(\alpha + \varphi + \varphi')}{\cos\varphi}$$
If $\varphi = \varphi'$, $\eta = \dfrac{\tan\alpha}{\tan(\alpha + 2\varphi)}$
14. A collar bearing or simply a collar is provided at any position along the shaft and bears the axial load on a mating surface.
15. When the axial load is taken by the end of the shaft which is inserted in a recess to bear the thrust, it is called a *pivot bearing* or simply a *pivot*. It is also known as *footstep bearing*.
16. In uniform pressure theory, pressure is assumed to be uniform over the surface area.
17. For uniform wear over an area, the intensity of pressure varies inversely proportional to the elementary areas and the product of the normal pressure and the corresponding radius is constant. Pressure intensity p at a radius r of the collar,
$$p = \frac{F}{2\pi r(R_o - R_i)}$$
18. For flat collars, friction torque is
$$T = \frac{2\mu F(R_o^3 - R_i^3)}{3(R_o^2 - R_i^2)} \text{ with uniform pressure theory}$$
$$= \frac{\mu F}{2}(R_o^2 + R_i^2) \text{ with uniform wear theory}$$
19. For conical collars, friction torque is increased by $1/\sin\alpha$ times from that for flat collars.
20. Expressions for torque in case of pivots can directly be obtained from the expressions for collars by inserting the values $R_i = 0$ and $R_o = R$.
21. To be on safer side, friction torque in clutches is calculated on the basis of uniform wear theory and in bearings on the basis of uniform pressure theory.
22. A clutch is a device used to transmit the rotary motion of one shaft to another when desired. The axes of the two shafts are coincident.
23. In a multi-plate clutch, the number of frictional linings and the metal plates is increased which increases the capacity of the clutch to transmit torque.
24. Ball and roller bearings are known as *anti-friction bearings*.
25. The property of a lubricant to form a layer of molecular thickness (adsorbed film) on a metallic surface is known as its *oiliness*.
26. Greasy or boundary friction occurs in heavily loaded, slow-running bearings.
27. A circle drawn with μr as radius is known as the *friction circle* of the journal.
28. Friction couple (torque) = $Wr\mu$
29. During rotation, the positions of the crank where the reaction at the crankshaft bearing and the friction axis of the connecting rod become aligned in the same straight line are known as *dead centre positions*.
30. For a journal rotating in a bearing under the film lubrication conditions, the frictional resistance is proportional to the area, the viscosity of the lubricant, the speed and is independent of the pressure and the materials of the journal and the bearing.

Exercises

1. What is friction? Is it a blessing or curse? Justify your answer giving examples.
2. What are various kinds of friction? Discuss each in brief.
3. What are the laws of solid dry friction?
4. Define the terms *coefficient of friction* and *limiting angle of friction*.
5. Deduce an expression for the efficiency of an inclined plane when a body moves
   (i) up a plane
   (ii) down a plane
6. Find expression for the screw efficiency of a square thread. Also, determine the condition for maximum efficiency.
7. Show that the reversal of the nut is avoided if the efficiency of square thread is less than 50% (approximately).
8. What is a wedge? Deduce an expression for its efficiency.
9. What are uniform pressure and uniform wear theories? Deduce expressions for the friction torque considering both the theories for a flat collar.
10. In what way are the expressions for the friction

torque of a conical collar changed from that for a flat collar? In what way are they modified for pivots.

11. Do you recommend the uniform pressure theory or uniform wear theory for the friction torque of a bearing? Explain.
12. What is a clutch? Make a sketch of a single-plate clutch and describe its working.
13. Explain the working of a multi-plate clutch with the help of a neat sketch.
14. Though cone clutches provide high frictional torque, yet they have become obsolete. Why?
15. Find a relation for the frictional torque acting on a centrifugal clutch.
16. Write a short note on anti-friction bearings.
17. Explain the terms *adsorbed film* and *oiliness* in case of greasy friction.
18. Explain the terms *friction circle, friction couple* and *friction axis*.
19. How is the correct tangent to the friction circle for correct friction axis of a slider-crank mechanism decided when the friction at the journals is considered?
20. What do you mean by film friction? State its laws.
21. Describe the working of a *Mitchell thrust bearing*.
22. A body on a rough horizontal surface requires a force of 240 N inclined at 25° just to pull it and 280 N just to push it at the same angle. Determine the weight of the body and the coefficient of friction.

    (1825 N, 0.126)
23. A force of 2.4 kN parallel to the plane surface is required to just move a body up an inclined plane, the angle of inclination being 8°. When the angle of inclination is increased to 12°, the force required increases to 3 kN. Determine the weight of the body and the coefficient of friction.

    (8.935 kW, 0.1307)
24. A power screw driven by an electric motor moves a nut in a horizontal plane when a force of 80 kN at a speed of 6 mm/s is applied. The screw is of single thread of 8-mm pitch and 48-mm major diameter. Determine the power of the motor if the coefficient of friction at the screw threads is 0.1.

    (1316.6 W)
25. Two wagons are coupled by using a turn buckle with right and left-hand single start threads. The mean diameter and the thread pitch are 48 mm and 10 mm respectively. The coefficient of friction between the screw and the nut is 0.14. Determine the work done in drawing the two wagons closer through a distance of 220 mm against a steady load of 3 kN. Also, find the additional work done if the load is increased to 8 kN over a distance of 300 mm.

    (1884 N.m, 3139.5 N.m)
26. A load of 12 kN is to be raised by means of a hand wheel with a threaded boss to act as a nut. The vertical screw is of single start square threads of 40-mm mean diameter and 10 mm pitch. The mean diameter of the bearing surface of the boss is 80 mm. The tangential force applied by each hand to the wheel is 120 N. If the coefficient of friction for the screw is 0.14 and for the bearing surface, it is 0.16, determine the suitable diameter of the hand wheel.

    (1.084 m)
27. A screw jack is used to raise a load of 5 tonnes (1 tonne = 9.81 kN). The pitch of single-start square threads used for the screw is 24 mm. The mean diameter is 72 mm. Determine the force to be applied at the end of 1.2 m long handle when the load is lifted with constant velocity and rotate with the spindle. Take $\mu = 0.2$. Also find the mechanical efficiency of the jack.

    (552.2 N; 33.9%)
28. Find the load that can be lifted by applying a force of 220 N at the end of a 500-mm long lever of screw jack using single-start square threads. The load does not rotate with the spindle and is carried on a swivel head having a bearing of 100 mm diameter. The pitch of the threads is 10 mm and the root diameter is 50 mm. Coefficient of friction between nut and thread is 0.18 and between spindle and swivel head is 0.15.

    Find also the efficiency of the jack.

    (7.8 kN; 11.29%)
29. Determine the mechanical efficiency of a wedge used to raise loads if the angle of wedge is 20° and the coefficient of friction is 0.2 between the frame and the wedge and 0.15 between the slider and the guide. The height of the guide is 120 mm and its lower end is 45 mm above the lower point of the axis of the slider which has a width of 50 mm. (38%)
30. The shaft of a collar thrust bearing rotates at 200 rpm and carries an end thrust of 10 tonnes. The outer and the inner diameters of the bearing are 480 mm and 280 mm respectively. If the power lost in friction is not to exceed 8 kW, determine the coefficient of friction of the lubricant of the bearing.

    (0.02)

31. A shaft carrying a load of 12 tonnes and running at 120 rpm has a number of collars integral with it. Shaft diameter is 240 mm and the external diameter of the collars is 360 mm. Intensity of uniform pressure is 400 kN/m² and the coefficient of friction is 0.06. Determine the power absorbed in overcoming the friction and the number of collars required.

(13.49 kW; 6)

32. A single-plate clutch, having two active surfaces, transmits 10 kW of power and the maximum torque developed is 120 N.m. Axial pressure is not to exceed 100 kN/m². Outer diameter of the friction plate is 1.3 times the inner diameter. Determine these diameters and the axial force exerted by the springs. Assume uniform wear and take coefficient of friction as 0.25.

(207 mm, 269mm)

33. The engine of an automobile is rated to give 80 kW at 1800 rpm with a maximum torque of 550 N.m. Design a dry single-plate clutch assuming the outer radius of the friction plate to be 1.2 times the inner radius. The coefficient of friction is 0.25 and the intensity of pressure between the plates is not to exceed 80 kN/m². Six springs are used to provide axial force necessary to engage the clutch and each spring has a stiffness of 50 N/mm. Find the initial compression in the springs and dimensions of the friction plate.

(15.49 mm)

34. A multiple disc clutch has 6 active friction surfaces. The power transmitted is 20 kW at 400 rpm. Inner and outer radii of the friction surfaces are 90 and 120 mm respectively. Assuming uniform wear with a coefficient of friction 0.3, find the maximum axial intensity of pressure between the discs.

(148.9 kN/m²)

35. Determine the axial force required to engage a cone clutch transmitting 25 kW of power at 750 rpm. Average friction diameter of the cone is 400 mm and average pressure intensity is 60 kN/m². Semi-cone angle is 10° and coefficient of friction 0.25. Also, find the width of the friction cone.

(2.672 kN; 84.4 mm)

36. Show that the torque transmitted by a cone clutch when intensity of pressure is uniform is given by

$$\frac{2\mu W}{3\sin\alpha}\left[\frac{r^3_o - r^3_i}{r^2_o - r^2_i}\right]$$

where W = axial load

μ = coefficient of friction of contact surfaces

α = semi-cone angle

r_o = outer radius of contact surface

r_i = inner radius of contact surface

In a cone clutch with semi-cone angle of 15°, maximum and minimum radii of the contact surface are 120 mm and 80 mm respectively. The speed is 800 rpm and the maximum allowable normal pressure is 150 kN/m². Determine the axial load and the power transmitted taking the coefficient of friction as 0.3.

(3.77 kN; 37.097 kW)

37. The crank of a steam engine is 250 mm long and the length of the connecting rod is 1 m. The steam pressure at the cross-head is 350 kN/m². The diameters of the journals at the crankshaft, crankpin and the crosshead are 180 mm, 140 mm and 100 mm respectively. The piston diameter is 200 mm and the coefficient of friction between the crosshead and guide is 0.079 and at the journal, it is 0.06. Determine the reduction in the turning moment available at the crankshaft due to friction of the crosshead guides and the journal at the moment when the crank has rotated through 45° from the inner-dead centre.

(58.7 Nm)

9 BELTS, ROPES AND CHAINS

Introduction

Usually, power is transmitted from one shaft to another by means of belts, ropes, chains and gears, the salient features of which are as follows:

1. Belts, ropes and chains are used where the distance between the shafts is large. For small distances, gears are preferred.
2. Belts, ropes and chains are flexible type of connectors, i.e., they are bent easily.
3. The flexibility of belts and ropes is due to the property of their materials whereas chains have a number of small rigid elements having relative motion between the two elements.
4. Belts and ropes transmit power due to friction between them and the pulleys. If the power transmitted exceeds the force of friction, the belt or rope slips over the pulley.
5. Belts and ropes are strained during motion as tensions are developed in them.
6. Owing to slipping and straining action, belts and ropes are not positive type of drives, i.e., their velocity ratios are not constant. On the other hand, chains and gears have constant velocity ratios.

This chapter deals with the power transmission by belts, ropes and chains. Power transmission by gears will be dealt in the next chapter. The belts used may be flat or of V shape. The selection of belt drive depends on the speeds of the velocity ratio, power to be transmitted, space available and the service conditions. A flat belt is used for light and moderate power transmission whereas for moderate to huge power transmission, more than one V belt or rope on pulleys with a number of grooves is used.

9.1 BELT AND ROPE DRIVES

To transmit power from one shaft to another, pulleys are mounted on the two shafts. The pulleys are then connected by an endless belt or rope passing over the pulleys. The connecting belt or rope is kept in tension so that motion of one pulley is transferred to the other without slip. The speed of the driven shaft can be varied by varying the diameters of the two pulleys.

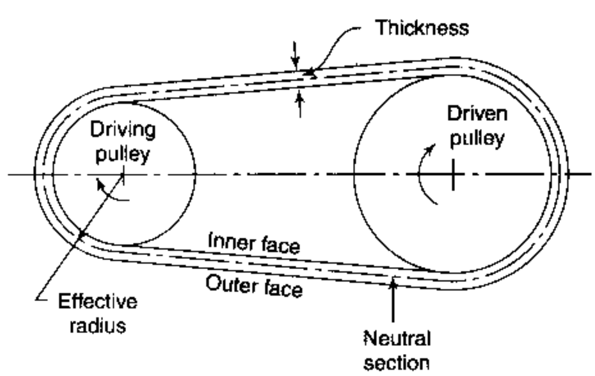


Fig. 9.1

For an unstreched belt mounted on the pulleys, the outer and the inner faces become engaged in tension and compression respectively (Fig. 9.1). In between there is a neutral section which has no tension or compression. Usually, this is considered at half the thickness of the belt. The effective radius of rotation of a pulley is obtained by adding half the belt thickness to the radius of the pulley.

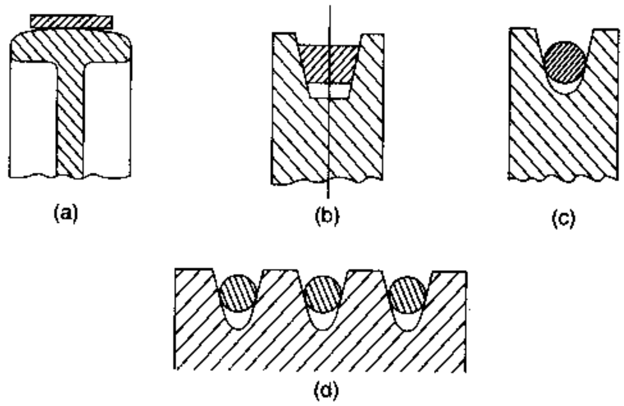


Fig. 9.2

Belt Drive A belt may be of rectangular section, known as a *flat belt* [Fig. 9.2(a)] or of trapezoidal section, known as a *V-belt* [Fig. 9.2(b)]. In case of a flat belt, the rim of the pulley is slightly *crowned* which helps to keep the belt running centrally on the pulley rim. The groove on the rim of the pulley of a V-belt drive is made deeper to take the advantage of the wedge action. The belt does not touch the bottom of the groove. Owing to wedging action, V-belts need little adjustment and transmit more power, without slip, as compared to flat belts. Also, a multiple V-belt system, using more than one belt in the two pulleys, can be used to increase the power transmitting capacity. Generally, these are more suitable for shorter centre distances.

Some advantages of V-belts are

- Positive drive as slip between belt and pulley is negligible
- No joint troubles as V-belts are made endless
- Operation is smooth and quite
- High velocity ratio up to 10 can be obtained
- Due to wedging action in the grooves, limiting ratio of tensions is higher and thus, more power transmission
- Multiple V-belt drive increases the power transmission manifold
- May be operated in either direction with tight side at the top or bottom
- Can be easily installed and removed.

Disadvantages of V-belts are

- Cannot be used for large centre distances
- Construction of pulleys is not simple
- Not as durable as flat belts
- Costlier as compared to flat belts.

Rope Drive For power transmission by ropes, grooved pulleys are used [Fig. 9.2(c)]. The rope is gripped on its sides as it bends down in the groove reducing the chances of slipping. Pulleys with several grooves can also be employed to increase the capacity of power transmission [Fig. 9.2(d)]. These may be connected in either of the two ways:

1. Using a continuous rope passing from one pulley to the other and back again to the same pulley in the next groove, and so on.
2. Using one rope for each pair of grooves.

The advantage of using continuous rope is that the tension in it is uniformly distributed. However, in case of belt failure, the whole drive is put out of action. Using one rope for each groove poses difficulty in tightening the ropes to the same extent but with the advantage that the system can continue its operation even if a rope fails. The repair can be undertaken when it is convenient.

Rope drives are, usually, preferred for long centre distances between the shafts, ropes being cheaper as compared to belts. These days, however, long distances are avoided and thus, the use of ropes has been limited.

9.2 OPEN- AND CROSSED-BELT DRIVES

1. Open-Belt Drive

An open belt drive is used when the driven pulley is desired to be rotated in the same direction as the driving pulley as shown in Fig. 9.1.

Generally, the centre distance for an open-belt drive is 14 to 16 m. If the centre distance is too large, the belt whips vibrate in a direction perpendicular to the direction of motion. For very shorter centre distances, the belt slip increases. Both these phenomena limit the use of belts for power transmission.

While transmitting power, one side of the belt is more tightened (known as tight side) as compared to the other (known as slack side). In case of horizontal drives, it is always desired that the tight side is at the lower side of two pulleys. This is because the sag of the belt will be more on the upper side than the lower side. This slightly increases the angles of wrap of the belt on the two pulleys than if the belt had been perfectly straight between the pulleys. In case the tight side is on the upper side, the sag will be greater at the lower side, reducing the angle of wrap, and slip could occur earlier. This ultimately affects the power to be transmitted.

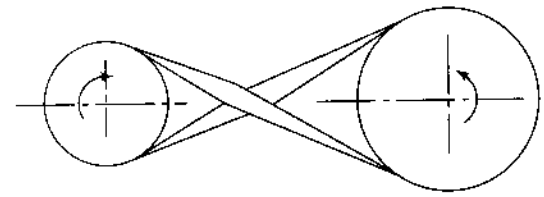


Fig. 9.3

2. Crossed-Belt Drive

A crossed-belt drive is adopted when the driven pulley is to be rotated in the opposite direction to that of the driving pulley (Fig. 9.3).

A crossed-belt drive can transmit more power than an open-belt drive as the angle of wrap is more. However, the belt has to bend in two different planes and it wears out more.

9.3 ACTION OF BELTS ON PULLEYS

To transmit torque from one shaft to another effectively, it is necessary that the belt does not slip over the pulley. To understand the action of belts on pulleys, consider a belt resting on the rim of a pulley. The belt is wrapped round the pulley to subtend an angle θ at its centre. The belt is tightened on the pulley by applying an equal amount of pull to the two ends of the belt. This results in an initial tension **T** on each end of the belt [Fig. 9.4(a)].

Now, if a small torque or tangential force **F** in the clockwise direction is applied to the driving pulley, it tends to rotate the belt with it. But if the motion of the belt is resisted, the pulley will have a tendency to slip over the belt. Considering the equilibrium of the pulley, it can be said that the slipping of the pulley is prevented by a frictional force **F′** (=**F**) acting in the counter-clockwise

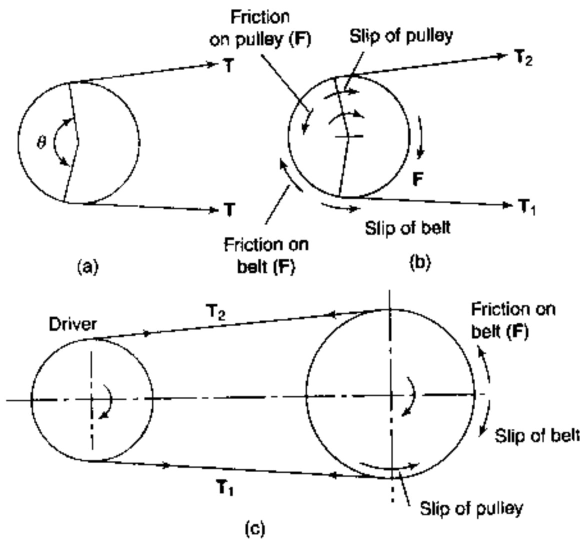


Fig. 9.4

direction. Similarly, considering the equilibrium of the belt, it can be said that the belt is prevented from slipping in the counter-clockwise direction (opposite to that of pulley) due to a frictional force equal to F in the clockwise direction (opposite to that on pulley). The frictional force in the belt results in an increased tension on one end of the belt (T_1) and a decrease in tension on the other (T_2) such that $(T_1 - T_2) = F$ [Fig. 9.4(b)].

If the tangential force on the pulley in increased, at one stage, it will just cause relative motion (slip) between the belt and the pulley. This will indicate that the frictional force has reached the limiting value and the force **F** on the pulley should not be increased further.

Now, consider a belt and pulley arrangement as shown in Fig. 9.4(c). Here, a belt with an initial tension T passes over driving and driven pulleys. On the driving side, the belt would rotate with the pulley with no relative slip between the two as long as the tangential force on the pulley, **F** ($=T_1 - T_2$) is less than the frictional force. On the driven side, the pulley rotates with the belt in the clockwise direction which means that the pulley can have a tendency to slip back in the counter-clockwise direction. This implies that the belt will have a tendency to slip over the pulley in the clockwise direction meaning a counter-clockwise frictional force equal to $(T_1 - T_2)$.

9.4 VELOCITY RATIO

Velocity ratio is the ratio of speed of the driven pulley to that of the driving pulley.

Let N_1 = rotational speed of the driving pulley
N_2 = rotational speed of the driven pulley
D_1 = diameter of the driving pulley
D_2 = diameter of the driven pulley
t = thickness of the belt.

Neglecting any slip between the belt and the pulleys and also considering the belt to be inelastic,
Speed of belt on driving pulley = speed of belt on driven pulley

$$\left(D_1 + 2\frac{t}{2}\right)N_1 = \left(D_2 + 2\frac{t}{2}\right)N_2$$

or velocity ratio (VR) $= \dfrac{N_2}{N_1} = \dfrac{D_1 + t}{D_2 + t}$ (9.1)

9.5 SLIP

The effect of slip is to decrease the speed of the belt on the driving shaft, and to decrease the speed of the driven shaft.

Let ω_1 = angular velocity of the driving pulley
ω_2 = angular velocity of the driven pulley
S_1 = percentage slip between the driving pulley and the belt
S_2 = percentage slip between the driven pulley and the belt
S = total percentage slip,

Peripheral speed of driving pulley $= \omega_1\left(\dfrac{D_1 + t}{2}\right)$

Speed of belt on the driving pulley $= \left[\omega_1\left(\dfrac{D_1 + t}{2}\right)\right]\left(\dfrac{100 - S_1}{100}\right)$

This is also the speed of the belt on the driven pulley.
Peripheral speed of driven pulley

$$= \left[\omega_1\left(\frac{D_1+t}{2}\right)\right]\left(\frac{100-S_1}{100}\right)\left(\frac{100-S_2}{100}\right)$$

As S is the total percentage slip,

$$\text{Peripheral speed of driven shaft} = \left[\omega_1\left(\frac{D_1+t}{2}\right)\right]\left(\frac{100-S}{100}\right)$$

$$\left[\omega_1\left(\frac{D_1+t}{2}\right)\right]\left(\frac{100-S_1}{100}\right)\left(\frac{100-S_2}{100}\right) = \left[\omega_1\left(\frac{D_1+t}{2}\right)\right]\left(\frac{100-S}{100}\right)$$

or $$\frac{(100-S_1)(100-S_2)}{100\times 100} = \frac{100-S}{100}$$

or $$(100-S_1)(100-S_2) = 100\,(100-S)$$

or $$10\,000 - 100\,S_2 - 100\,S_1 + S_1 S_2 = 10\,000 - 100\,S$$

or $$100\,S = 100\,S_1 + 100\,S_2 - S_1 S_2$$

or $$S = S_1 + S_2 - 0.01\,S_1 S_2 \qquad (9.2)$$

Effect of slip is to reduce the velocity ratio,

$$\text{VR} = \frac{N_2}{N_1} = \left(\frac{D_1+t}{D_2+t}\right)\left(\frac{100-S}{100}\right) \qquad (9.3)$$

Also, it is to be remembered that slip will first occur on the pulley with smaller angle of lap, i.e., on the smaller pulley.

Example 9.1 ***A shaft runs at 80 rpm and drives another shaft at 150 rpm through belt drive. The diameter of the driving pulley is 600 mm. Determine the diameter of the driven pulley in the following cases:***

(i) Neglecting belt thickness
(ii) Taking belt thickness as 5 mm
(iii) Assuming for case (ii) a total slip of 4%
(iv) Assuming for case (ii) a slip of 2% on each pulley

Solution $N_1 = 80$ rpm $\quad D_1 = 600$ mm
$N_2 = 150$ rpm

(i) $$\frac{N_2}{N_1} = \frac{D_1}{D_2} \quad \text{or} \quad \frac{150}{80} = \frac{600}{D_2}$$

or $D_2 = \underline{320\text{ mm}}$

(ii) $$\frac{N_2}{N_1} = \frac{D_1+t}{D_2+t} \quad \text{or} \quad \frac{150}{80} = \frac{600+5}{D_2+5}$$

$D_2 = 317.7$ mm

(iii) $$\frac{N_2}{N_1} = \frac{D_1+t}{D_2+t}\left(\frac{100-S}{100}\right)$$

or $$\frac{150}{80} = \left(\frac{600+5}{D_2+5}\right)\left(\frac{100-4}{100}\right)$$

$D_2 = 304.8$ mm

(iv) $$\frac{N_2}{N_1} = \frac{D_1+t}{D_2+t}\left(\frac{100-S}{100}\right)$$

where $S = S_1 + S_2 - 0.01 S_1 S_2$
$= 2 + 2 - 0.01 \times 2 \times 2$
$= 3.96$

$$\frac{150}{80} = \left(\frac{600+5}{D_2+5}\right)\left(\frac{100-3.96}{100}\right)$$

$D_2 = \underline{304.9\text{ mm}}$

9.6 MATERIAL FOR BELTS AND ROPES

Choice of materials for the belts and ropes is influenced by climate or environmental conditions along with the service requirements. The common materials are as given below:

1. Flat Belts

Usual materials for flat belts are leather, canvas, cotton and rubber. These belts are used to connect shafts up to 8–10 m apart with speeds as high as 22 m/s.

Leather belts are made from 1.2 to 1.5 m long strips. The thickness of a belt may be increased by cementing the strips together. The belts are specified by the number of layers, i.e., single, double or triple ply. The leather belts are cleaned and dressed periodically with suitable oils to keep them soft and flexible.

Fabric belts are made by folding cotton or canvas layers to three or more layers and stitching together. The belts are made waterproof by impregnating with linseed oil. These are mostly used in belt conveyors and farm machinery.

Rubber belts are very flexible and are destroyed quickly on coming in contact with heat, grease or oil. Usually, these are made endless. Rubber belts are used in paper and saw mills as these can withstand moisture.

2. V-Belts

These are made of rubber impregnated fabric with the angle of V between 30 to 40 degrees. These are used to connect shafts up to 4 m apart. Speed ratios can be up to 7 to 1 and belt speeds up to 24 m/s.

3. Ropes

The materials for ropes are cotton, hemp, manila or wire. Ropes may be used to connect shafts up to 30 m apart with operating speed less than 3 m/s.

Hemp and *manila* fibres are rough and thus, the ropes made from such materials are not very flexible. Manila ropes are stronger as compared to hemp ropes. Generally, the rope fibres are lubricated with tar, tallow or graphite to prevent sliding of fibres when the ropes are bent over the pulleys. The cotton ropes are soft and smooth and do not require lubrication. These are not as strong and durable as manila ropes.

Wire ropes are used when the power transmitted is large over long distances, may be up to 150 m such as cranes, conveyors, elevators, etc. Wire ropes are lighter in weight, have silent operation, do not fail suddenly, more reliable and durable, less costly and can withstand shock loads.

9.7 CROWING OF PULLEYS

As mentioned is Section 9.2, the rim of the pulley of a flat-belt drive is slightly crowned to prevent the slipping off the belt from the pulley. The crowing can be in the form of conical surface or a convex surface.

Assume that somehow a belt comes over the conical portion of the pulley and takes the position as shown in Fig. 9.5(a), i.e., its centre line remains in a plane, the belt will touch the rim surface at its one edge only. This is impractical. Owing to the pull, the belt always tends to stick to the rim surface. The belt also has a lateral stiffness. Thus, a belt has to bend in the way shown in Fig. 9.5(b) to be on the conical surface of the pulley.

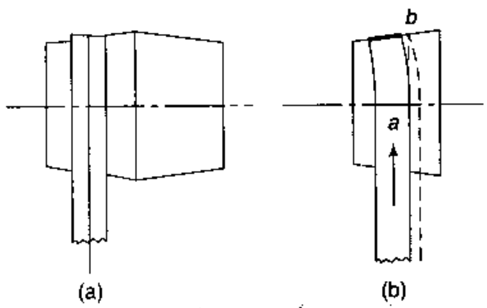


Fig. 9.5

Let the belt travel in the direction of the arrow. As the belt touches the cone, the point a on it tends to adhere to the cone surface due to pull on the belt. This means as the pulley will take a quarter turn, the point a on the belt will be carried to b which is towards the mid-plane of the pulley than that previously occupied by the edge of the belt. But again, the belt cannot be stable on the pulley in the upright position and has to bend to stick to the cone surface, i.e., it will occupy the position shown by dotted lines.

Thus, if a pulley is made up of two equal cones or of convex surface, the belt will tend to climb on the slopes and will thus, run with its centre line on the mid-plane of the pulley.

The amount of crowing is usually 1/96 of the pulley face width.

9.8 TYPES OF PULLEYS

1. Idler Pulleys

With constant use, the belt is permanently stretched a little in length. This reduces the initial tension in the belt leading to lower power transmission capacity. However, the tension in the belt can be restored to the original value by using an arrangement shown in Fig. 9.6(a).

A bell-crank lever, hinged on the axis of the smaller pulley, supports adjustable weights on its one arm and the axis of a pulley on the other. The pulley is free to rotate on its axis and is known as *idler pulley*. Owing to weights on one arm of the lever, the pulley exerts pressure on the belt increasing the tension and the angle of contact. Thus, life of the belt is increased and power capacity is restored to the previous value.

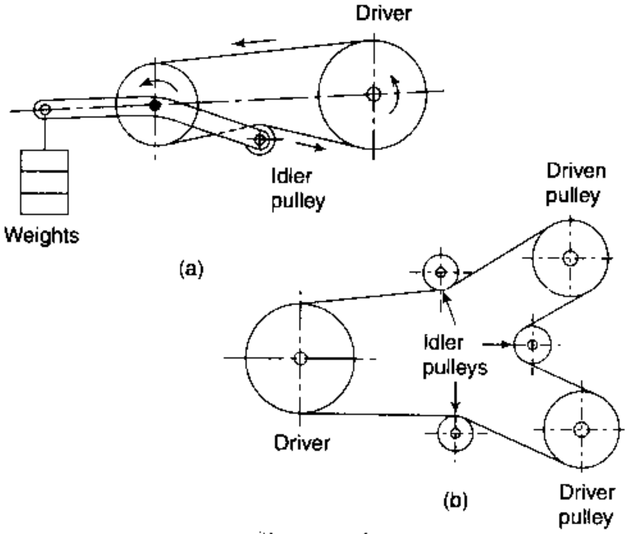


Fig. 9.6

The pressure force on the belt can be varied by changing the weights on the arm of the lever.

Motion of one shaft can be transmitted to two or more than two shafts by using a number of idler pulleys. This has been illustrated in Fig. 9.6(b).

A compound belt drive

2. Intermediate Pulleys

When it is required to have large velocity ratios, ordinarily, the size of the larger pulley will be quite big. However, by using an *intermediate* (or *countershaft*)

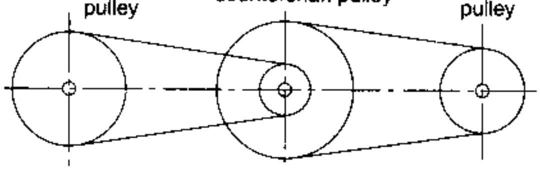


Fig. 9.7

pulley, the size can be reduced as shown in Fig. 9.7. This type of drive is also referred as *compound belt drive*.

3. Loose and Fast Pulleys

Many times, it is required to drive several machines from a single main shaft. In such cases, some arrangement to link or delink a machine to or from the main shaft has to be incorporated as all the machines may not be operating simultaneously. The arrangement, usually, provided is that of using a loose pulley along with a fast pulley (Fig. 9.8).

The *fast pulley* is keyed to the shaft and rotates with it at the same speed and thus, transmits power. A *loose pulley* is not keyed to the shaft and thus, is unable to transmit any power. Whenever, a machine is to be driven, the belt is mounted on the fast pulley and when it is not required to transmit any power, the belt is pushed on to the loose pulley placed adjacent to the fast pulley.

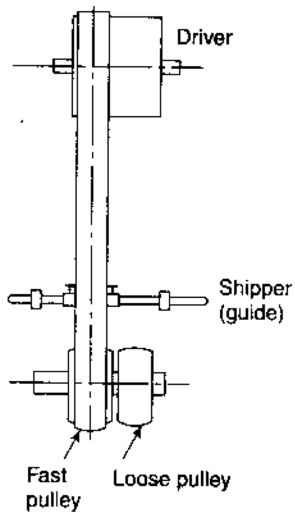


Fig. 9.8

4. Guide Pulleys

A *guide pulley* is used to connect two non-parallel shafts in such a way that they may run in either direction, and still make the pulleys deliver the belt properly in accordance with the law of belting as shown in Fig. 9.9(a) (refer Sec. 9.9 for the law of belting). A guide pulley can also be used to connect even the intersecting shafts as shown in Fig. 9.9(b).

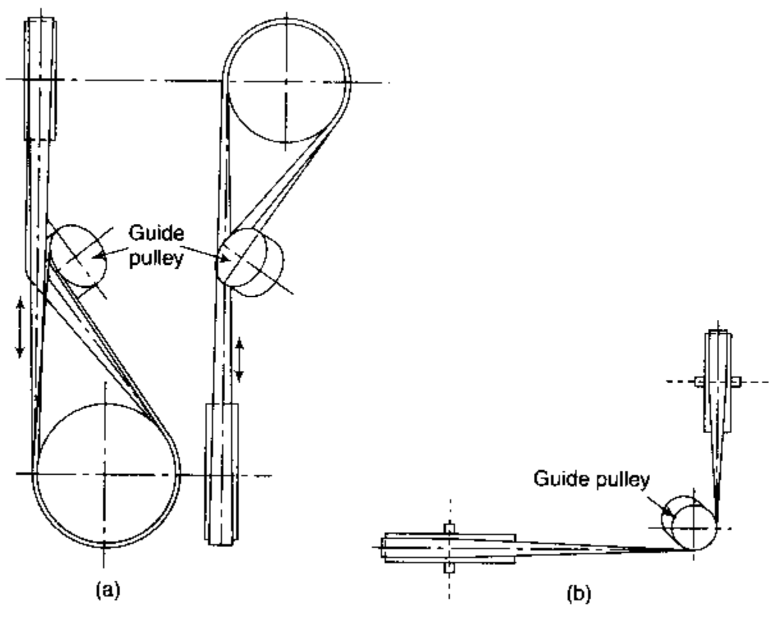


Fig. 9.9

9.9 LAW OF BELTING

The law of belting states that the centre line of the belt when it approaches a pulley must lie in the mid plane of that pulley. However, a belt leaving a pulley may be drawn out of the plane of the pulley. In other words, the plane of a pulley must contain the point at which the belt leaves the other pulley.

By following this law, non-parallel shafts may be connected by a flat belt. In Fig. 9.10, two shafts with two pulleys are at right angles to each other. It can be observed that the centre line of the belt approaching the larger pulley lies in its plane which is also true for the smaller pulley. Also, the points at which the belt leaves a pulley are contained in the plane of the other pulley.

It should also be observed that it is not possible to operate the belt in the reverse direction without violating the law of belting. Thus, in case of non-parallel shafts, motion is possible only in one direction. Otherwise, the belt is thrown off the pulley. However, it is possible to run a belt in either direction on the pulleys of two non-parallel or intersecting shafts with the help of guide pulleys (refer to Sec. 9.8). The law of belting is still satisfied.

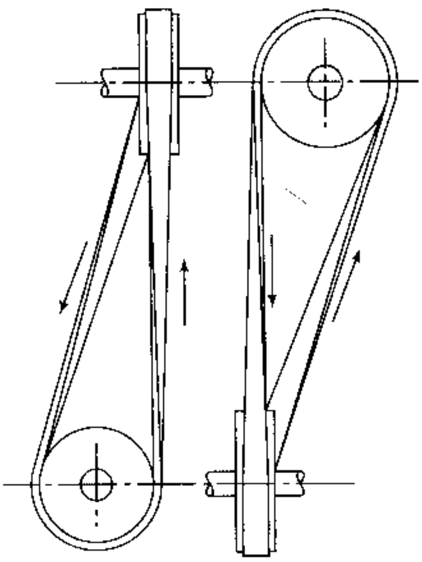


Fig. 9.10

9.10 LENGTH OF BELT

1. Open Belt

Let A and B be the pulley centres and CD and EF, the common tangents to the two pulley circles (Fig. 9.11). Total length of the belt comprises

(a) the length in contact with the smaller pulley
(b) the length in contact with the larger pulley
(c) the length not in contact with either pulley

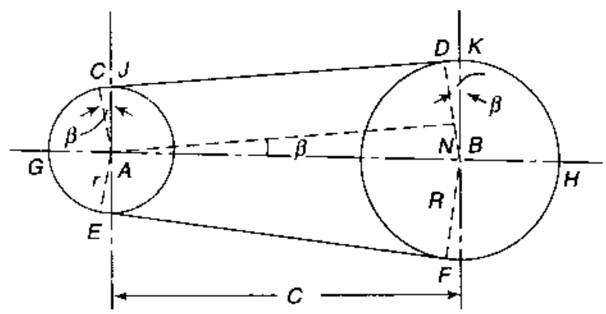


Fig. 9.11

Let L_o = length of belt for open belt drive

r = radius of smaller pulley

R = radius of larger pulley

C = Centre distance between pulleys

β = angle subtended by each common tangent (CD or EF) with AB, the line of centres of pulleys.

Draw AN parallel to CD so that $\angle BAN = \beta$ and $BN = R - r$

As CD is tangent to two circles, AC and BD both are perpendicular to CD or AN.

Now, $AB \perp BK$ and $AN \perp BD$.

$\therefore \quad \angle DBK = \angle NAB = \beta$

Similarly, as $BA \perp AJ$, $NA \perp AC$

$\angle CAJ = \angle NAB = \beta$

$$L_o = 2\,[\text{Arc } GC + CD + \text{arc } DH]$$

$$= 2\left[\left(\frac{\pi}{2} - \beta\right) r + AN + \left(\frac{\pi}{2} + \beta\right) R\right]$$

$$= 2\left[\left(\frac{\pi}{2} - \beta\right) r + C\cos\beta + \left(\frac{\pi}{2} + \beta\right) R\right]$$

$$= \pi(R + r) + 2\beta(R - r) + 2C\cos\beta \tag{9.4}$$

This relation gives the exact length of belt required for an open belt drive. In this relation,

$$\beta = \sin^{-}\left(\frac{R - r}{C}\right) \tag{9.5}$$

An approximate relation for the length of belt can also be found in terms of R, r and C eliminating β, if β is small, i.e., if the difference in radii of the two pulleys is small and the centre distance is large.

For small angle of β, $\sin\beta \approx \beta$

$$\therefore \qquad \beta = \frac{R - r}{C}$$

and

$$\cos\beta = \sqrt{1 - \sin^2\beta}$$

$$= (1 - \sin^2\beta)^{1/2}$$

$$= \left(1 - \frac{1}{2}\sin^2\beta + \ldots\ldots\right) \qquad \text{[By binomial theorem]}$$

or

$$\cos\beta = \left(1 - \frac{1}{2}\beta^2\right) = 1 - \frac{1}{2}\left(\frac{R - r}{C}\right)^2$$

$$L_o = \pi(R + r) + 2\left(\frac{R - r}{C}\right)(R - r) + 2C\left[1 - \frac{1}{2}\left(\frac{R - r}{C}\right)^2\right]$$

$$= \pi(R + r) + 2\frac{(R - r)^2}{C} + 2C - \frac{2C}{2}\frac{(R - r)^2}{C^2}$$

$$= \pi(R + r) + 2\frac{(R - r)^2}{C} - \frac{(R - r)^2}{C} + 2C$$

$$= \pi(R + r) + \frac{(R - r)^2}{C} + 2C \tag{9.6}$$

2. Crossed-Belt

As before, let A and B be the pulley centres and CD and EF, the common tangents (crossed) to the two pulley circles (Fig. 9.12).

Draw AN parallel to CD meeting BD produced at N so that $\angle BAN = \beta$

We have, $\angle CAJ = \angle DBK = \beta$

Let L_c = length of belt for crossed-belt drive

Then

$$L_c = 2[\text{Arc } GC + CD + \text{Arc } DH]$$

$$= 2\left[\left(\frac{\pi}{2}+\beta\right)r + AN + \left(\frac{\pi}{2}+\beta\right)R\right]$$

$$= 2\left[\left(\frac{\pi}{2}+\beta\right)r + C\cos\beta + \left(\frac{\pi}{2}+\beta\right)R\right]$$

$$= (\pi + 2\beta)(R + r) + 2C\cos\beta \tag{9.7}$$

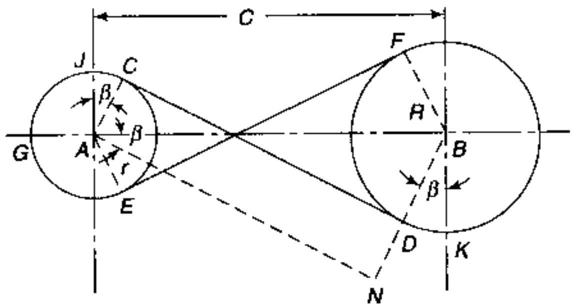


Fig. 9.12

This is the exact length of a crossed-belt drive where

$$\beta = \sin^{-1}\left(\frac{R+r}{C}\right) \tag{9.8}$$

For small angle of β, $\sin\beta \approx \beta$

$$\therefore \qquad \beta = \frac{R+r}{C}$$

and

$$\cos\beta = \left(1 - \frac{1}{2}\beta^2\right) = 1 - \frac{1}{2}\left(\frac{R+r}{C}\right)^2$$

$$L_c = \left[\pi + 2\left(\frac{R+r}{C}\right)\right](R+r) + 2C\left[1 - \frac{1}{2}\left(\frac{R+r}{C}\right)^2\right]$$

$$= \pi(R+r) + 2\frac{(R+r)^2}{C} + 2C - \frac{(R+r)^2}{C}$$

$$= \pi(R+r) + \frac{(R+r)^2}{C} + 2C \tag{9.9}$$

This is an approximate relation for the length in terms of R, r and C.

It can be noted that the length of belt depends only on the sum of the pulley radii and the centre distance in case of crossed-belt drive whereas it depends on the sum as well as the difference of the pulley radii apart from the centre distance in case of open-belt drive.

Example 9.2 *Two parallel shafts, connected by a crossed belt, are provided with pulleys 480 mm and 640 mm in diameters. The distance between the centre lines of the shafts is 3 m. Find by how much the length of the belt should be changed if it is desired to alter the direction of rotation of the driven shaft.*

Solution $R = 320$ mm

$C = 3$ m

$r = 240$ mm

For cross belt

$$\beta = \sin^{-}\left(\frac{R+r}{C}\right) = \sin^{-}\left(\frac{0.32+0.24}{3}\right) = \sin^{-} 0.1867$$

$= 10°45'$ or 0.1878 rad

$\cos\beta = 0.9825$

$L_c = (\pi + 2\beta)(R + r) + 2C\cos\beta$

$= (\pi + 2 \times 0.1878)(0.32 + 0.24) + 2 \times 3 \times 0.9825$

$= \underline{7.865\ \text{m}}$

For open belt

$$\beta = \sin^{-}\left(\frac{R-r}{C}\right) = \sin^{-}\left(\frac{0.32-0.24}{3}\right) = \sin^{-} 0.0267$$

$$= 1°32'$$

As the angle is very small, the approximate relation can be used.

$$L_o = \pi(R+r) + \frac{(R-r)^2}{C} + 2C$$

$$= \pi(0.32+0.24) + \frac{(0.32-0.24)^2}{3} + 2 \times 3$$

$$= \underline{7.761\ \text{m}}$$

The length of the belt should be reduced by

$L_c - L_o = 7.865 - 7.761 = 0.104$ m or $\underline{104\ \text{mm}}$

CONE (STEPPED) PULLEYS

Many times, it is required to run the driven shaft at different speeds whereas the driving shaft runs at constant speed which is the speed of the motor. This is facilitated by using a pair of cone or stepped pulleys (Fig. 9.13). A cone pulley has different sets of pulley radii to give varying speeds of the driven shaft. The radii of different steps are so chosen that the same belt can be used at different sets of the cone pulleys.

Let n = speed of the driving shaft (constant)

N_n = speed of the driven shaft when the belt is on nth step

r_n = radius of the nth step of the driving pulley

R_n = radius of the nth step of the driven pulley

The subscript n denotes 1, 2, 3, ... n.

The ratio of speeds of driving to driven shaft is inversely proportional to the ratio of their pulley radii, i.e.,

$$\frac{N_1}{n} = \frac{r_1}{R_1} \qquad \text{(i)}$$

Thus, to get speed N_1 of the driven shaft from the first pair of steps of the cone pulleys, dimensions of r_1 and R_1 can be chosen convenient to the design.

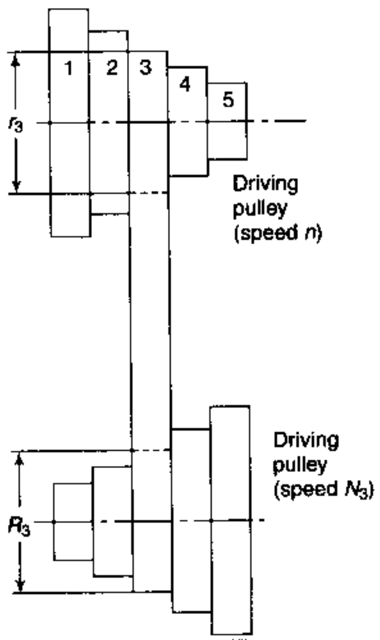


Fig. 9.13

For the second pair of steps,

$$\frac{N_2}{n} = \frac{r_2}{R_2}$$

Again some convenient dimensions of r_2 and R_2 can be chosen according to the ratio of N_2/n. Similarly, for other pair of steps also, the same procedure can be adopted. For the nth pair,

$$\frac{N_n}{n} = \frac{r_n}{R_n} \quad \text{(ii)}$$

However, it is always desired that the same belt is used on all the pairs of steps of the cone pulley. To fulfil this condition, the length of the belt has to be the same for all pairs of steps, i.e.,

$$L_1 = L_2 = \ldots\ldots\ldots\ldots = L_n$$

or

$$(R_1 + r_1) + \frac{(R_1 - r_1)^2}{C} + 2C = (R_2 + r_2) + \frac{(R_2 - r_2)^2}{C} + 2C = \ldots\ldots$$

$$= (R_n + r_n) + \frac{(R_n - r_n)^2}{C} + 2C$$

or

$$(R_1 + r_1) + \frac{(R_1 - r_1)^2}{C} = (R_2 + r_2) + \frac{(R_2 - r_2)^2}{C} = \ldots\ldots$$

$$= (R_n + r_n) + \frac{(R_n - r_n)^2}{C} \quad (9.10)$$

As R_1 and r_1 have already been chosen for the first pair and ratios $r_2/R_2, \ldots, r_n/R_n$ are known, their values can be easily calculated.

Also, it is usual practice to have speeds of the driven shaft in geometrical progression and to make the driving and the driven cones similar.

Let K = ratio of progression of speed

Then $$\frac{N_2}{N_1} = \frac{N_3}{N_2} = \ldots = \frac{N_n}{N_{n-1}} = K$$

$\therefore$ $$N_2 = KN_1$$

$$N_3 = KN_2 = K^2 N_1$$

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

$$N_n = KN_{n-1} = K^{n-1} N_1$$

To obtain these speeds of the driven shaft, the ratio of the radii of different pairs of pulleys can be obtained as under:

$$N_n = K^{n-1} N_1$$

$$= K^{n-1} n \frac{r_1}{R_1} \quad \text{[from (i)]}$$

or $$\frac{N_n}{n} = K^{n-1} \frac{r_1}{R_1}$$

or $$\frac{r_n}{R_n} = K^{n-1} \frac{r_1}{R_1} \quad \text{[from (ii)]}$$

Thus, $\frac{r_2}{R_2} = K \frac{r_1}{R_1}; \frac{r_3}{R_3} = K^2 \frac{r_1}{R_1}$, and so on.

If the driving and the driven pulleys are to be made similar,

$$\frac{r_n}{R_n} = \frac{R_1}{r_1} \text{ or } K^{n-1}\frac{r_1}{R_1} = \frac{R_1}{r_1} \text{ or } \left(\frac{R_1}{r_1}\right)^2 = K^{n-1} \text{ or } \frac{R_1}{r_1} = \sqrt{K^{n-1}} \tag{9.11}$$

This gives the ratio R_1/r_1. After deciding its value, ratio r_2/R_2, r_3/R_3, etc., can be obtained and then from the relation for the length of belt, the values of r_2, R_2, and r_3, R_3, etc., can be obtained.

If cone pulleys are being used for a crossed-belt drive, it is very easy to obtain the dimensions of the radii of different pairs of steps. In this case, the length of the belt is same if the sums of the radii of different pairs of steps are constant for a given centre distance between the pulleys, i.e.,

$$R_1 + r_1 = R_2 + r_2 = \ldots\ldots = R_n + r_n \tag{9.12}$$

Example 9.3 *Design a set of stepped pulleys to drive a machine from a countershaft that runs at 220 rpm. The distance between centres of the two sets of pulleys is 2 m. The diameter of the smallest step on the countershaft is 160 mm. The machine is to run at 80, 100 and 130 rpm and should be able to rotate in either direction.*

Solution: As the driven shaft is to rotate in either direction, both the cases of a crossed-belt and an open-belt are to be considered.

(*i*) ***For Crossed-belt System***

The smallest step on the countershaft will correspond to the biggest step on the machine shaft (or the minimum speed of the machine shaft).

$n_1 = n_2 = n_3 = 220$ rpm $\qquad r_1 = 80$ mm

$N_1, N_2, N_3 = 80, 100, 130$ rpm respectively

(*a*) *For First Step*

$$\frac{R_1}{r_1} = \frac{n_1}{N_1} \text{ or } \frac{R_1}{80} = \frac{220}{80} \text{ or } R_1 = \underline{220 \text{ mm}}$$

(*b*) *For Second Step*

$$\frac{R_2}{r_2} = \frac{n_2}{N_2} = \frac{220}{100} \text{ or } R_2 = 2.2r_2$$

Also $\quad R_2 + r_2 = R_1 + r_1$

$2.2r_2 + r_2 = 220 + 80$

$3.2r_2 = 300$

$r_2 = \underline{93.75 \text{ mm}}$

$R_2 = 93.7 \times 2.2 = \underline{206.3 \text{ mm}}$

(*c*) *For third Step*

$$\frac{R_3}{r_3} = \frac{220}{130} \text{ or } R_3 = 1.69r_3$$

Also $\quad R_3 + r_3 = R_1 + r_1$

$1.69\, r_3 + r_3 = 220 + 80 = 300$

$r_3 = \underline{111.5 \text{ mm}}$

$R_3 = 111.5 \times 1.69 = \underline{188.5 \text{ mm}}$

(*ii*) ***For Open-belt System***

(*a*) *For First Step* $r_1 = 80$ mm $R_1 = 220$ mm as before

(*b*) *For Second Step*

$$\pi(R_2 + r_2) + \frac{(R_2 - r_2)^2}{C} = \pi(R_1 + r_1) + \frac{(R_1 - r_1)^2}{C}$$

or $$\pi(2.2r_2 + r_2) + \frac{(2.2r_2 - r_2)^2}{2} = \pi(0.22 + 0.08) + \frac{(0.22 - 0.08)^2}{2}$$

$$10.05r_2 + \frac{1.44}{2}r_2^2 = 0.9523$$

$r_2^2 + 13.958\, r_2 = 1.323$

$(r_2 + 6.979)^2 = 1.323 + (6.979)^2$

$= 50.029 = (7.073)^2$

$r_2 = 7.073 - 6.979 = 0.094$ m or 94 mm

$R_2 = 2.2 \times 94 = \underline{206.8 \text{ mm}}$

(*c*) *For Third Step*

$$\pi(1.69r_3 + r_3) + \frac{(1.69r_3 - r_3)^2}{2} = 0.9523$$

or $$8.451r_3 + \frac{0.476}{2}r_3^2 = 0.9523$$

or $r_3^2 + 35.508r_3 = 4.001$

or $(r_3^2 + 17.754)^2 = 4.001 + (17.754)^2$

$= 319.206 = (17.866)^2$

or $r_3 = 17.866 - 17.754 = 0.112$ m or 112 mm

$R_3 = 112 \times 1.69 = \underline{189.3 \text{ mm}}$

RATIO OF FRICTION TENSIONS

1. Flat Belt

Let $\mathbf{T_1}$ = tension on tight side
$\mathbf{T_2}$ = tension on slack side
θ = angle of lap or contact of the belt over the pulley
μ = coefficient of friction between the belt and the pulley

Consider a short length of belt subtending an angle $\delta\theta$ at the centre of the pulley (Fig. 9.14).

Let **R** = normal (radial) reaction between the element length of belt and the pulley
T = tension on slack side of the element
$\delta\mathbf{T}$ = increase in tension on tight side than that on slack side
$\mathbf{T} + \delta\mathbf{T}$ = tension on tight side of the element

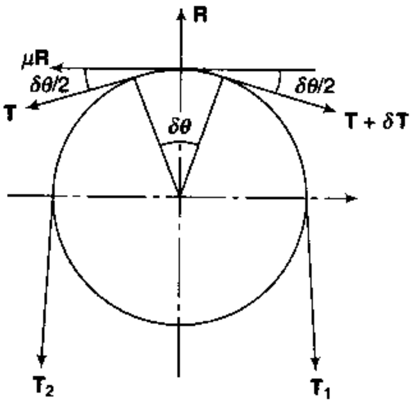


Fig. 9.14

Tensions **T** and ($\mathbf{T} + \delta\mathbf{T}$) act in directions perpendicular to the radii drawn at the ends of the elements. The friction force $\mu\mathbf{R}$ will act tangentially to the pulley rim resisting the slipping of the elementary belt on the pulley.

Resolving the forces in the tangential direction,

$$\mu R + T\cos\frac{\delta\theta}{2} - (T + \delta T)\cos\frac{\delta\theta}{2} = 0$$

As $\delta\theta$ is small,

$$\cos\frac{\delta\theta}{2} \approx 1$$

$\therefore$
$$\mu R + T - T - \delta T = 0 \quad \text{or} \quad \delta T = \mu R \qquad \text{(i)}$$

Resolving the forces in the radial direction,

$$R - T\sin\frac{\delta\theta}{2} - (T + \delta T)\sin\frac{\delta\theta}{2} = 0$$

As $\delta\theta$ is small,
$$\sin\frac{\delta\theta}{2} \approx \frac{\delta\theta}{2}$$

Thus,
$$R - T\frac{\delta\theta}{2} - T\frac{\delta\theta}{2} - \frac{\delta T\,\delta\theta}{2} = 0$$

Neglecting product of two small quantities,

$$R = T\,\delta\theta \qquad \text{(ii)}$$

Inserting this value of R in (i),

$$\delta T = \mu . T\delta\theta \quad \text{or} \quad \frac{\delta T}{T} = \mu\delta\theta$$

Integrating between proper limits,

$$\int_{T_2}^{T_1} \frac{dT}{T} = \int_0^{\theta} \mu \, d\theta$$

or $$\log_e \frac{T_1}{T_2} = \mu\theta$$

or $$\frac{T_1}{T_2} = e^{\mu\theta} \quad (9.13)$$

It is to be noted that the above relation is valid only when the belt is on the point of slipping on the pulleys.

2. V-Belt or Rope

In case of a V-belt or rope, there are two normal reactions as shown in Fig. 9.15 so that the radial reaction is equal to 2**R** sin α.

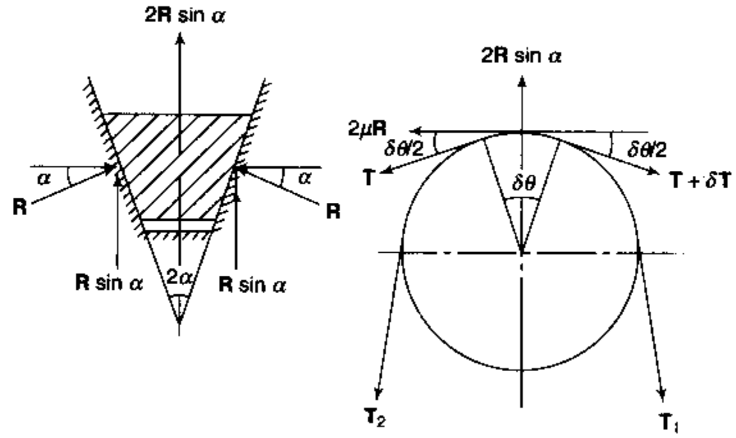


Fig. 9.15

Thus, total frictional force = $2(\mu R) = 2\mu R$.

Resolving the forces tangentially,

$$2\mu R + T\cos\frac{\delta\theta}{2} - (T + \delta T)\cos\frac{\delta\theta}{2} = 0$$

For small angle of $\delta\theta$,

$$\cos\frac{\delta\theta}{2} \approx 1$$

$\therefore$ $$\delta T = 2\mu R \quad \text{(iii)}$$

Resolving the forces radially,

$$2R\sin\alpha - T\sin\frac{\delta\theta}{2} - (T + \delta T)\sin\frac{\delta\theta}{2} = 0$$

As $\delta\theta$ is small, $\sin\frac{\delta\theta}{2} \approx \frac{\delta\theta}{2}$.

$$2R\sin\alpha - T\frac{\delta\theta}{2} - T\frac{\delta\theta}{2} = 0$$

or $$R = \frac{T\delta\theta}{2\sin\alpha} \qquad \text{(iv)}$$

From (iii) and (iv),

$$\delta T = 2\mu\frac{T\delta\theta}{2\sin\alpha}$$

or $$\frac{\delta T}{T} = \frac{\mu\delta\theta}{\sin\alpha}$$

Integrating between proper limits,

$$\int_{T_2}^{T_1}\frac{dT}{T} = \int_0^{\theta}\frac{\mu\,d\theta}{\sin\alpha}$$

or $$\log_e\frac{T_1}{T_2} = \frac{\mu\theta}{\sin\alpha}$$

or $$\frac{T_1}{T_2} = e^{\mu\theta/\sin\alpha} \qquad (9.14)$$

The expression is similar to that for a flat-belt drive except that μ is replaced by $\mu/\sin\theta$, i.e., the coefficient of friction is increased by $1/\sin\theta$. Thus, the ratio T_1/T_2 is far greater in case of V-belts and ropes for the same angle of lap θ and coefficient of friction μ.

Again, it is to be noted that the above expression is derived on the assumption that the belt is on the point of slipping.

9.13 POWER TRANSMITTED

Let T_1 = tension on the tight side
T_2 = tension on the slack side
v = linear velocity of the belt
P = power transmitted

Then,

P = Net force × Distance moved/second
$= (T_1 - T_2) \times v$ (9.15)

This relation gives the power transmitted irrespective of the fact whether the belt is on the point of slipping or not. If it is, the relationship between T_1 and T_2 for a flat belt is given by $T_1/T_2 = e^{\mu\theta}$. If it is not, no particular relation is available to calculate T_1 and T_2.

Example 9.4 ***A belt runs over a pulley of 800-mm diameter at a speed of 180 rpm. The angle of lap is 165° and the maximum tension in the belt is 2 kN. Determine the power transmitted if the coefficient of friction between the belt and the pulley is 0.3.***

Solution $T_1 = 2000$ N $\quad d = 0.8$ m

$N = 180$ rpm $\quad \mu = 0.3$

$\theta = 165° = 165 \times \pi/180 = 2.88$ rad

$$v = \frac{\pi dN}{60} = \frac{\pi \times 0.8 \times 180}{60} = 7.54 \text{ m/s}$$

$$\frac{T_1}{T_2} = e^{\mu\theta} = e^{0.3 \times 2.88} = 2.37 \text{ or } T_1 = 2.37T_2$$

or $\quad 2000 = 2.37\, T_2$ or $T_2 = 843$ N

and $\quad P = (T_1 - T_2)\, v = (2000 - 843) \times 7.54$

$= 8724$ W or 8.724 kW

Example 9.5 ***A casting weighs 6 kN and is freely suspended from a rope which makes 2.5 turns round a drum of 200-mm diameter. If the drum rotates at 40 rpm, determine the force required by a man to pull the rope from the other end of the rope. Also, find the power to raise the casting. The coefficient of friction is 0.25.***

Solution $T_1 = 6000$ N $\quad d = 0.2$ m

$N = 40$ rpm $\quad \mu = 0.25$

$\theta = 2.5 \times 2\pi = 15.7$ rad

$$v = \frac{\pi dN}{60} = \frac{\pi \times 0.2 \times 40}{60} = 0.419 \text{ m/s}$$

$$\frac{T_1}{T_2} = e^{\mu\theta} = e^{0.25 \times 15.7} = 50.8 \text{ or } T_1 = 50.8T_2$$

or $\quad 6000 = 50.8\, T_2$ or $T_2 = 118$ N

and $\quad P = (T_1 - T_2)\, v = (6000 - 118) \times 0.419$

$= 2464$ W or 2.464 kW

Example 9.6 ***A belt drive transmits 8 kW of power from a shaft rotating at 240 rpm to another shaft rotating at 160 rpm. The belt is 8 mm thick. The diameter of the smaller pulley is 600 mm and the two shafts are 5 m apart. The coefficient of friction is 0.25. If the maximum stress in the belt is limited to 3 N/mm², find the width of the belt for (i) an open belt drive, and (ii) a cross-belt drive.***

Solution Speed of the driving pulley, $N_1 = 240$ rpm

Speed of the driven pulley, $N_2 = 160$ rpm

Thus, smaller pulley is the driver and

$d = 600$ mm

$r = 300$ mm; $P = 8$ kW; $C = 5$ m; $\mu = 0.25$;

$t = 8$ mm

$$D = \frac{240}{160} \times 600 = 900 \text{ mm or } R = 450 \text{ mm}$$

$$v = \frac{\pi dN_1}{60} = \frac{\pi \times 0.6 \times 240}{60} = 7.54 \text{ m/s}$$

$P = (T_1 - T_2)\, v$

or $\quad 8000 = (T_1 - T_2) \times 7.54$

or $\quad T_1 - T_2 = 1061 \qquad$ (i)

(i) Open-belt drive

In designing the belt drive, the angle of contact on the smaller pulley has to be considered as it is the lesser of the two angles of contact.

Now, angle of contact on the smaller pulley,

$$\theta = \pi - 2\beta = \pi - 2\sin^{-1}\left(\frac{R-r}{C}\right)$$

$$\text{or } \theta = \pi - 2\sin^{-}\left(\frac{450-300}{5000}\right)$$

$$= \pi - 3.438° = \pi - 0.06 = 3.082 \text{ rad}$$

$$\text{Also } \frac{T_1}{T_2} = e^{\mu\theta}$$

$$\text{or } \frac{T_1}{T_2} = e^{0.25 \times 3.082} = 2.161 \text{ or } T_1 = 2.161T_2 \qquad \text{(ii)}$$

From (i) and (ii),

$2.161\, T_2 - T_2 = 1061$

$T_2 = 914$ N

$T_1 = 1975$ N

The maximum tension, $T_1 = \sigma.b.t$

or $1975 = 3 \times b \times 8$ or $b = 82.3$ mm

(ii) Cross-belt drive

$$\theta = \pi + 2\beta = \pi + 2\sin^{-1}\left(\frac{R+r}{C}\right)$$

or $\theta = \pi + 2\sin^{-}\left(\frac{450+300}{5000}\right)$

$= \pi + 17.254° = \pi + 0.301 = 3.443$ rad

Also $\frac{T_1}{T_2} = e^{\mu\theta}$

or $\frac{T_1}{T_2} = e^{0.25\times 3.443} = 2.365$ or $T_1 = 2.365 T_2$ (iii)

From (i) and (iii),

$2.365T_2 - T_2 = 1061$

$T_2 = 777$ N

$T_1 = 1838$ N

The maximum tension, $T_1 = \sigma.b.t$

or $1838 = 3b \times 8$ or $b = 76.6$ mm

Example 9.7 ***A 100-mm wide and 10-mm thick belt transmits 5 kW of power between two parallel shafts. The distance between the shaft centres is 1.5 m and the diameter of the smaller pulley is 440 mm. The driving and the driven shafts rotate at 60 rpm and 150 rpm respectively. The coefficient of friction is 0.22. Find the stress in the belt if the two pulleys are connected by (i) an open belt, and (ii) a cross belt. Take μ = 0.22.***

Solution Speed of the driving pulley, $N_1 = 60$ rpm

Speed of the driven pulley, $N_2 = 150$ rpm

Thus, smaller pulley is the driven pulley and $d = 440$ mm

$P = 5$ kW; $b = 100$ mm; $C = 1.5$ m; $t = 10$ mm; $\mu = 0.22$; $r = 220$ mm;

$$v = \omega_2\left(r + \frac{t}{2}\right) = \frac{2\pi N_2}{60}\left(r + \frac{t}{2}\right)$$

$$= \frac{2\times\pi\times 150}{60}\left(220 + \frac{10}{2}\right)$$

$= 3535$ mm/s or 3.535 m/s

$P = (T_1 - T_2)v$ or $5000 = (T_1 - T_2) \times 3.535$

or $T_1 - T_2 = 1414.5$ N (i)

__*(i) Open-belt Drive*__

Angle of contact on the smaller pulley,

$$\theta = \pi - 2\beta = \pi - 2\sin^{-1}\left(\frac{R-r}{C}\right)$$

$$= \pi - 2\sin^{-1}\left(\frac{220\times 150/60 - 220}{1500}\right)$$

$= \pi - 25.4°$

$= \pi - 0.443$ (β is to be in radians)

$= 2.698$ rad.

$\frac{T_1}{T_2} = e^{\mu\theta}$

or $\frac{T_1}{T_2} = e^{0.22\times 2.698} = 1.81$ or $T_1 = 1.81 T_2$ (ii)

From (i) and (ii), $T_1 - T_2 = 1414.5$

∴ $1.81T_2 - T_1 = 1414.5$

or $0.81T_2 = 1414.5$

$T_2 = 1746.3$ N and $T_1 = 1746.3 \times 1.81 = 3160.8$ N

∴ stress in the belt,

$$\sigma_t = \frac{T_1}{b\times t} = \frac{3160.8}{100\times 10} = \underline{3.16 \text{ N/mm}^2}$$

__*(ii)* Cross-belt Drive__

$$\theta = \pi + 2\beta = \pi + 2\sin^{-1}\left(\frac{R+r}{C}\right)$$

or $\theta = \pi + 2\sin^{-1}\left(\frac{220\times 150/60 + 220}{1500}\right)$

$= \pi + 61.8°$

or $\theta = \pi + 1.08 = 4.22$ rad

$\frac{T_1}{T_2} = e^{0.22\times 4.22} = 2.53$

$T_1 - T_2 = 3160.8 - 1746.3 = 1414.5$ (iii)

From (i) and (iii), $2.53T_2 - T_1 = 1414.5$

$T_2 = 924.5$ N

$T_1 = 924.5 \times 2.53 = 2339$ N

$$\sigma_t = \frac{2339}{100\times 10} = \underline{2.339 \text{ N/mm}^2}$$

9.16 CENTRIFUGAL EFFECT ON BELTS

While in motion, as a belt passes over a pulley, the centrifugal effect due to its own weight tends to lift the belt from the pulley. Owing to symmetry, the centrifugal force produces equal tensions on the two sides of the belt, i.e., on the tight side as well as on the slack side.

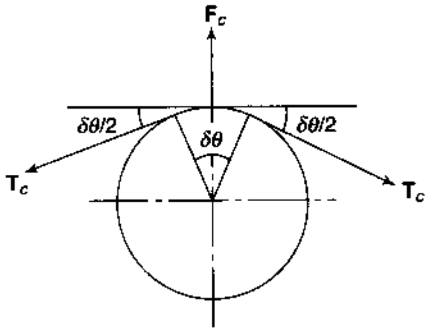


Fig. 9.16

Consider a short element of belt (Fig. 9.16).

Let m = mass per unit length of belt.

T_c = centrifugal tension on tight and slack sides of element

F_c = centrifugal force on the element

r = radius of the pulley

v = velocity of the belt

$\delta\theta$ = angle of lap of the element over the pulley

F_c = mass of element × acceleration

= (length of element × mass per unit length) × acceleration

$$= (r\delta\theta \times m) \times \frac{v^2}{r}$$

$$= mv^2\delta\theta \qquad \text{(i)}$$

Also,

$$F_c = 2T_c \sin\frac{\delta\theta}{2}$$

As $\delta\theta$ is small,

$$\sin\frac{\delta\theta}{2} \approx \frac{\delta\theta}{2}$$

$$F_c = 2T_c\frac{\delta\theta}{2}$$

$$= T_c\delta\theta \qquad \text{(ii)}$$

From (i) and (ii),

$$T_c\delta\theta = mv^2\delta\theta$$

or

$$T_c = mv^2 \qquad (9.16)$$

Thus, centrifugal tension is independent of the tight and slack side tensions and depends only on the velocity of the belt over the pulley.

Also,

$$\text{Centrifugal stress in the belt} = \frac{\text{Centrifugal tension}}{\text{area of cross section of belt}} = \frac{T_c}{a}$$

Total tension on tight side = friction tension + centrifugal tension

$$T = T_1 + T_c \tag{9.17}$$

Total tension of slack side = $T_2 + T_c$

It can be shown that the power transmitted is reduced if centrifugal effect is considered for a given value of the total tight side tension T.

(a) *Centrifugal Tension Considered*

Friction tension on tight side = $T - T_c = T_1$

Let T_2 be the friction tension on the slack side.

Then $\dfrac{T_1}{T_2} = e^{\mu\theta} = k$, a constant

or $T_2 = \dfrac{T_1}{k}$

and power, $P = (T_1 - T_2)v = \left(T_1 - \dfrac{T_1}{k}\right)v = T_1\left(1 - \dfrac{1}{k}\right)v$

(b) *Centrifugal Tension Neglected*

Friction tension on tight side = T

Let T_2' be the friction tension on slack side.

$\dfrac{T_1}{T_2'} = e^{\mu\theta} = k$, or $T_2' = \dfrac{T}{k}$

Power, $P = (T_1 - T_2')v = \left(T - \dfrac{T}{k}\right)v = T\left(1 - \dfrac{1}{k}\right)v$

As T_1 is lesser than T, power transmitted is less when centrifugal force is taken into account.

9.15 MAXIMUM POWER TRANSMITTED BY A BELT

If it is desired that a belt transmits maximum possible power, two conditions must be fulfilled simultaneously.

1. Larger tension must reach the maximum permissible value for the belt.
2. The belt should be on the point of slipping. i.e., maximum frictional force is developed in the belt.

Now,

$$P = (T_1 - T_2)v = T_1\left(T_1 - \frac{T_2}{T_1}\right)v = T_1\left(1 - \frac{1}{e^{\mu\theta}}\right)v = T_1 kv$$

where $k = 1 - \dfrac{1}{e^{\mu\theta}}$ = constant

or $$P = (T - T_c)\,kv$$
$$= kTv - kmv^2\,v = kTv - kmv^3$$

The maximum tension T in the belt should not exceed the permissible limit. Hence, treating T as constant and differentiating the power with respect to v and equating the same equal to zero, we get

$$\frac{dP}{dv} = kT - 3\,kmv^2 = 0$$

or
$$T = 3mv^2 = 3T_c$$

or
$$T_c = \frac{T}{3} \tag{9.18}$$

Therefore, for maximum power transmission, centrifugal tension in the belt must be equal to one-third of the maximum allowable belt tension and the belt should be on the point of slipping.

Also,
$$T_1 = T - T_c = T - \frac{T}{3} = \frac{2}{3}T$$

and
$$v_{max} = \sqrt{\frac{T}{3m}} \tag{9.19}$$

Example 9.8 ***An open-belt drive is required to transmit 10 kW of power from a motor running at 600 rpm. Diameter of the driving pulley is 250 mm. The speed of the driven pulley is 220 rpm. The belt is 12 mm thick and has a mass density of 0.001 g/mm². Safe stress in the belt is not to exceed 2.5 /mm². The two shafts are 1.25 m apart. The coefficient of friction is 0.25.***

Determine the width of the belt.

Solution Speed of the driving pulley, N_1 = 600 rpm

Speed of the driven pulley, N_2 = 220 rpm

Thus, smaller pulley is the driver and

d = 250 mm

P = 10 kW; t = 12 mm;

$\rho = 0.001$ g/mm² = 1000 kg/m² ; r = 125 mm;

C = 1.25 m; N_2 = 220 rpm ; $\mu = 0.25$;

$\sigma_t = 2.5$ N/mm² = 2.5×10^6 N/m²

To calculate the width of the belt, we need to know the maximum tension in the belt which is the sum of the tight side tension and the centrifugal tension,

i.e., $T = T_1 + T_2$

Calculation of T_1

$$P = (T_1 - T_2)v$$

where $v = \omega\left(r + \frac{t}{2}\right) = \frac{2\pi N}{60}\left(r + \frac{t}{2}\right)$

$$= \frac{2\pi \times 600}{60}\left(125 + \frac{12}{2}\right) = 8230 \text{ mm/s or } 8.23 \text{ m/s}$$

$\therefore$ $10\,000 = (T_1 - T_2) \times 8.23$

or $T_1 \cdot T_2 = 1215$

Also $\frac{T_1}{T_2} = e^{\mu\theta}$

where $\theta = \pi - 2\beta = \pi - 2\sin^{-1}\left(\frac{R_2 - R_1}{C}\right)$

or $\theta = \pi - 2\sin^{-}\left(\frac{125 \times 600/220 - 125}{1250}\right)$

or $\theta = \pi - 19.9° = \pi - 0.347 = 2.79$

$$\frac{T_1}{T_2} = e^{0.25 \times 2.79} = 2.01 \text{ or } T_1 = 2.01T_2 \qquad \text{(ii)}$$

From (i) and (ii),

$2.01\,T_2 - T_1 = 1215$

$T_2 = 1203$ N

$T_1 = 2418$ N

Calculation of T_c

$T_c = mv^2$

= mass per unit length × v^2

= volume per unit length × density × v^2

= (x-sectional area × length × density) × v^2

= (width × thickness × length × density) × v^2

= $b \times 0.012 \times 1 \times 1000 \times (8.23)^2$

= $(812.8b)$ N (b in m)

$T = T_1 + T_c = \sigma_t \times (b \times t)$

$2418 + 812.8b = 2.5 \times 10^6 \times b \times 0.012$

$29\,187\,b = 2418$

$b = 0.0828$ m or 82.8 mm

Example 9.9 *Two parallel shafts that are 3.5m apart are connected by two pulleys of 1-m and 400-mm diameters, the larger pulley being the driver runs at 220 rpm. The belt weighs 1.2 kg per metre length. The maximum tension in the belt is not to exceed 1.8 kN. The coefficient of friction is 0.28. Owing to slip on one of the pulleys, the velocity of the driven shaft is 520 rpm only. Determine the*

(i) torque on each shaft
(ii) power transmitted
(iii) power lost in friction
(iv) efficiency of the drive

Solution The larger pulley is the driver, $D = 1$ m; $N_1 = 220$ rpm;

$R = 500$ mm; $d = 400$ mm; $r = 200$ mm; $N_2 = 520$ rpm; $C = 3.5$ m; $\mu = 0.28$; $m = 1.2$ kg/m; $T = 1800$ N

$$v = \frac{\pi D N_1}{60} = \frac{\pi \times 1 \times 220}{60} = 11.52 \text{ m/s}$$

Also $T_c = mv^2 = 1.2 \times 11.52^2 = 159$ N

$\therefore$ tension on the tight side, $T_1 = T - T_c$
$= 1800 - 159 = 1641$ N

$$\text{Now, } \theta = \pi - 2\sin^{-1}\left(\frac{R-r}{C}\right)$$

$$= \pi - 2\sin^{-}\left(\frac{500-200}{3500}\right) = \pi - 9.834°$$

$$= \pi - 0.172 = 2.97 \text{ rad}$$

$$\text{Also } \frac{T_1}{T_2} = e^{\mu\theta} = e^{0.28 \times 2.97} = 2.297 \text{ or } T_1 = 2.297\,T_2$$

or $T_2 = 1641/2.297 = 714$ N

(i) Torque on larger pulley $= (T_1 - T_2)R$
$= (1641 - 714) \times 0.5 = 463.5$ N.m
Torque on smaller pulley $= (T_1 - T_2)r$
$= (1641 - 714) \times 0.2 = 185.4$ N.m

(ii) $P = (T_1 - T_2)v = (1641 - 714) \times 11.52$
$= 10679$ W $= 10.679$ kW

(iii) $\text{Power input} = \dfrac{2\pi N_1 T_1}{60} = \dfrac{2\pi \times 220 \times 463.5}{60}$
10 678 W...(T_1 is the torque)

$\text{Power output} = \dfrac{2\pi N_2 T_2}{60} = \dfrac{2\pi \times 520 \times 185.4}{60}$
$= 10\,096$ W...(T_2 is the torque)
Power loss = 10 678 − 10 096 = 582 W

(iv) $\text{Efficiency} = \dfrac{\text{Output power}}{\text{Input power}} = \dfrac{10096}{10678}$
$= 0.945$ or 9.45%

Example 9.10 *A V-belt drive with the following data transmits power from an electric motor to a compressor:*

Power transmitted	=	*100 kW*
Speed of the electric motor	=	*750 rpm*
Speed of the compressor	=	*300 rpm*
Diameter of compressor pulley	=	*800 mm*
Centre distance between pulleys	=	*1.5 m*
Maximum speed of the belt	=	*30 m/s*
Mass density of the belt	=	*900 kg/m³*
Cross-sectional area of belt	=	*350 mm²*
Allowable stress in the belt	=	*2.2 N/mm²*
Groove angle of the pulley	=	*38°*
Coefficient of friction	=	*0.28*

Determine the number of belts required and the length of each belt.

Solution Speed of driving pulley (electric motor), $N_1 = 750$ rpm

Speed of the driven pulley, $N_2 = 300$ rpm

Thus, larger pulley is the driven pulley and $D = 800$ mm

$$\therefore \quad \frac{d}{D} = \frac{N_2}{N_1} \text{ or } \frac{d}{800} = \frac{300}{750} \text{ or } d = 320 \text{ mm or}$$

$r = 160$ mm

Mass of belt/m length = area × length × density
$= 350 \times 10^{-6} \times 1 \times 900 = 0.315$ kg

Centrifugal tension, $T_c = mv^2 = 0.315 \times 30^2$
$= 283.5$ N

Maximum tension in the belt,
$T = \sigma \times \text{area} = 2.2 \times 350 = 770$ N
$T_1 = T - T_c = 770 - 283.5 = 486.5$ N

$$\text{Now, } \theta = \pi - 2\sin^{-1}\left(\frac{R-r}{C}\right)$$

$$= \pi - 2\sin^{-}\left(\frac{400-160}{1500}\right) = \pi - 18.4^\circ$$

$$= \pi - 0.32 = 2.82 \text{ rad}$$

Also, $\frac{T_1}{T_2} = e^{\mu\theta/\sin\alpha} = e^{0.28 \times 2.82/\sin 19^\circ}$

$= 11.3$ or $T_1 = 11.3T_2$

$\therefore \quad T_2 = 486.5/11.3 = 43.1$ N

$P = (T_1 - T_2)v = (486.5 - 43.1) \times 30$

$= 13300$ W or 13.3 kW

Number of belts

$$= \frac{\text{Total power transmitted}}{\text{power transmitted/belt}} = \frac{60}{13.3} = 4.51 \text{ or } \underline{5}$$

Using approximate relation for the length of the belt,

$$L_o = \pi(R+r) + \frac{(R-r)^2}{C} + 2C$$

$$= \pi(0.4+0.16) + \frac{(0.4-0.16)^2}{3} + 2\times1.5 = \underline{4.79 \text{ m}}$$

Example 9.11 ***Determine the maximum power transmitted by a V-belt drive having the included V-groove angle of 35°. The belt used is 18 mm deep with 18 mm maximum width and weighs 300 g per metre length. The angle of lap is 145° and the maximum permissible stress is 1.5 N/mm². Take coefficient of friction to be 0.2.***

Solution $m = 0.3$ kg/m; $\alpha = 35/2 = 17.5^\circ$; $b = 18$ mm; $t = 18$ mm; $\theta = 145^\circ = 2.53$ rad; $\mu = 0.2$;

Maximum tension in the belt $= \sigma.b.t.$

$= 1.5 \times 18 \times 18 = 486$ N

Under maximum power conditions,

$$T_1 = \frac{2}{3}T = \frac{2}{3} \times 486 = 324 \text{ N}$$

$T_c = T - T_1 = 486 - 324 = 162$ N

Also, $T_c = mv^2$ or $162 = 0.3\,v^2$ or $v = 23.2$ m/s

Now,

$$\frac{T_1}{T_2} = e^{\mu\theta/\sin\alpha} = e^{0.2\times2.53/\sin 17.5^\circ}$$

$= 5.38$ or $T_1 = 5.38T_2$

or $\quad T_2 = 324/5.38 = 60.2$ N

(ii) $P = (T_1 - T_2)\,v = (324 - 60.2) \times 23.2$

$= 6120$ W or 6.12 kW

Example 9.12 ***The grooves on the pulleys of a multiple-rope drive have an angle of 50° and accommodate ropes of 22 mm diameter having a mass of 0.8 kg per metre length for which a safe operating tension of 1200 N has been laid down. The two pulleys are of equal size. The drive is designed for maximum power conditions. Speed of both the pulleys is 180 rpm. Assuming coefficient of friction as 0.25, determine the diameters of the pulleys and the number of ropes when the power transmitted is 150 kW.***

Solution $T = 1200$ N $\quad P = 150\,000$ W

$m = 0.8$ kg/m length $\quad \theta = 180^\circ$ ....(two pulleys are of equal size)

$\alpha = \frac{50}{2} = 25^\circ \quad \mu = 0.25$

$N_1 = N_2 = 180$ rpm

Under maximum power conditions,

$$T_1 = \frac{2}{3}T = \frac{2}{3} \times 1200 = 800 \text{ N}$$

$T_c = T - T_1 = 1200 - 800 = 400$ N

Also $T_c = mv^2$ or $400 = 0.8\,v^2$ or $v = 22.36$ m/s

or $\quad v = \frac{\pi D_1 N_1}{60} = 22.36$

or $\quad \frac{\pi \times D_1 \times 180}{60} = 22.36$

$D_1 = \underline{2.37 \text{ m}}$ Also $D_2 = \underline{2.37 \text{ m}}$

$$\frac{T_1}{T_2} = e^{\mu\theta/\sin\alpha} = e^{0.25\times\frac{180\times\pi}{180}\frac{1}{\sin 25^\circ}} = 6.41$$

$$T_2 = \frac{T_1}{6.41} = \frac{800}{6.41} = 124.8 \text{ N}$$

$P = (T_1 - T_2)\,v.n.$

$150\,000 = (800 - 124.8) \times 22.36 \times n$

$n = 9.94$ say $\underline{10 \text{ ropes}}$

9.16 INITIAL TENSION

When a belt is first fitted to a pair of pulleys, an initial tension T_o is given to the belt when the system is stationary. When transmitting power, the tension on the tight side increases to T_1 and that on slack side decreases to T_2. If it is assumed that the material of the belt is perfectly elastic, i.e., the strain in the belt is proportional to stress in it and the total length of the belt remains unchanged, the tension on the tight side will increase by the same amount as the tension on the slack side decreases. If this change in the tension is δT then

tension on tight side, $T_1 = T_o + \delta T$

tension on slack side, $T_2 = T_o - \delta T$

$$\therefore \quad T_o = \frac{T_1 + T_2}{2}$$

= mean of the tight and the slack side tensions. (9.20)

Initial Tension with Centrifugal Tension

Total tension on tight side $= T_1 + T_c$

Total tension on slack side $= T_2 + T_c$

$$\therefore \quad T_o = \frac{(T_1 + T_c) + (T_2 + T_c)}{2}$$

$$= \frac{T_1 + T_2}{2} + T_c$$

or $\quad T_1 + T_2 = 2\,(T_o - T_c)$

Let $\quad \dfrac{T_1}{T_2} = e^{\mu\theta} = k$

Therefore,

$$kT_2 + T_2 = 2(T_o - T_c)$$

$$T_2 = \frac{2(T_o - T_c)}{k+1}$$

and

$$T_1 = \frac{2k(T_o - T_c)}{k+1} \qquad \text{(i)}$$

$$T_1 - T_2 = \frac{2k(T_o - T_c)}{k+1} - \frac{2(T_o - T_c)}{k+1}$$

$$= \frac{2(k-1)(T_o - T_c)}{k+1}$$

Power transmitted, $\quad P = (T_1 - T_2).v$

$$= \frac{2(k-1)(T_o - T_c)}{k+1}v = \frac{2(k-1)(T_o - mv^2)}{k+1}v$$

$$= \frac{2(k-1)(T_o v - mv^3)}{k+1}$$

To find the condition for maximum power transmission, differentiating this expression with respect to v and equating the same to zero, i.e.,

$$\frac{dP}{dv} = T_o - 3mv^2 = 0$$

$$T_o = 3mv^2$$

$$v = \sqrt{\frac{T_o}{3m}}$$

When the belt drive is started, $v = 0$ and Thus, $T_c = 0$,

$$T_1 = \frac{2kT_o}{k+1} \qquad \text{(ii)}$$

From (i) and (ii), it is evident that the maximum tension in the belt is more while starting the drive.

Example 9.13 ***The following data relate to a rope drive:***

Power transmitted	= *20 kW*
Diameter of pulley	= *480 mm*
Speed	= *80 rpm*
Angle of lap on smaller pulley	= *160°*
Number of ropes	= *8*
Mass of rope/m length	= *48 G^2 kg*
Limiting working tension	= *132 G^2 kN*
Coefficient of friction	= *0.3*
Angle of groove	= *44°*

If G is the girth of rope in m, determine the initial tension and the diameter of each rope.

Solution Power transmitted /rope = 20 000/8 = 2500 W

$$\text{Velocity of rope} = \frac{\pi DN}{60} = \frac{\pi \times 0.48 \times 80}{60} = 2.01 \text{ m/s}$$

Now, $P = (T_1 - T_2)\, v$

or $2500 = (T_1 - T_2) \times 2.01$

or $(T_1 - T_2) = 1244$ N (i)

Also, $\dfrac{T_1}{T_2} = e^{\mu\theta/\sin\alpha} = e^{0.3 \times \frac{160\times\pi}{180}\frac{1}{\sin 22°}} = 9.359$ (ii)

From (i) and (ii),

$9.359\, T_2 - T_2 = 1244$

$T_2 = 148.8$ N

$T_1 = 148.8 \times 9.359 = 1392.8$ N

$$\text{Initial tension} = \frac{T_1 + T_2}{2} = \frac{1392.8 + 148.8}{2} = 770.8 \text{ N}$$

Now, Working tension = Tension on tight side + Centrifugal tension

$= T_1 + mv^2$

$132\,000\, G^2 = 1392.8 + 48\, G^2 . (2.01)^2$

$131\,806\, G^2 = 1392.8$

$G^2 = 0.01056$

or $G = 0.1028$

Now girth (circumference) of rope

$= \pi d = 0.1028$

or $d = \underline{0.327 \text{ m}}$

Example 9.14 *2.5 kW of power is transmitted by an open-belt drive. The linear velocity of the belt is 2.5 m/s. The angle of lap on the smaller pulley is 165°. The coefficient of friction is 0.3.*

Determine the effect on power transmission in the following cases:

(i) Initial tension in the belt is increased by 8%

(ii) Initial tension in the belt is decreased by 8%

(iii) Angle of lap is increased by 8% by the use of an idler pulley, for the same speed and the tension on the tight side

(iv) Coefficient of friction is increased by 8% by suitable dressing to the friction surface of the belt

Solution $P = 2.5$ kW $\mu = 0.3$

$\theta = 165°$ $v = 2.5$ m/s

$P = (T_1 - T_2)v$

$$2500 = (T_1 - T_2) \times 2.5$$

$$T_1 - T_2 = 1000 \text{ N}$$

$$\frac{T_1}{T_2} = e^{\mu\theta} = e^{0.3 \times 165\pi/180} = 2.37$$

or $T_1 = 2.37T_2$

$2.37T_2 - T_2 = 1000$

or $T_2 = 729.9$ N

$T_1 = 729.9 \times 2.37 = 1729.9$ N

Initial tension,

$$T_0 = \frac{T_1 + T_2}{2} = \frac{1729.3 + 729.9}{2} = 1229.9 \text{ N}$$

(i) When initial tension is increased by 8%

$T'_0 = 1229.9 \times 1.08 = 1328.3$ N

or $\frac{T_1 + T_2}{2} = 1328.3$ or $T_1 + T_2 = 2656.6$

As μ and θ remain unchanged, $e^{\mu\theta}$ or $\frac{T_1}{T_2}$ is same.

$2.37\,T_2 + T_2 = 2556.6$

$T_2 = 788.3$ N

$T_1 = 1868.3$ N

$P = (T_1 - T_2)v = (1868.3 - 788.3) \times 2.5$

$= 2700$ W or 2.7 kW

$\therefore$ increase in power $= \frac{2.7 - 2.5}{2.5} = 0.08$ or 8%

(ii) When initial tension is decreased by 8%

$T'_0 = 1229.9 \times (1 - 0.08) = 1131.5$

or $\frac{T_1 + T_2}{2} = 1131.5$ or $T_1 + T_2 = 2263$

$3.37T_2 = 2263$

$T_2 = 671.5$ N

$T_1 = 1591.5$ N

$P = (1591.5 - 671.5) \times 2.5 = 2300$ W or 2.3 kW

$\therefore$ Decrease in power $= \frac{2.5 - 2.3}{2.5} = 0.08$ or 8%

(iii) $\frac{T_1}{T_2} = e^{\mu\theta}$

T_1 is the same as before whereas θ increases by 8%

$$\frac{1729.9}{T_2} = e^{0.3 \times \frac{165 \times 1.08 \times \pi}{180}} = 2.54$$

$T_2 = 680.5$ N

$P = (1729.9 - 680.5) \times 2.5 = 2624$ W

or 2.624 kW

$\therefore$ Increase in power

$= \frac{2.624 - 2.5}{2.5} = 0.0496$ or 4.96%

(iv) $\frac{T_1}{T_2} = e^{\mu\theta} = e^{0.3 \times 1.08 \times \frac{165 \times \pi}{180}} = 2.54$

or $T_1 = 2.54\,T_2$

$T_1 + T_2 = 1229.9 \times 2 = 2459.8$

$T_2 = 694.9$ N

$T_1 = 694.9 \times 2.54 = 1764.9$ N

$P = (1764.9 - 694.9) \times 2.5$

$= 2675$ W or 2.675 kW

$\therefore$ Increase in power

$= \frac{2.675 - 2.5}{2.5} = 0.07$ or 7%

Example 9.15 *In a belt drive, the mass of the belt is 1 kg/m length and its speed is 6 m/s. The drive transmits 9.6 kW of power. Determine the initial tension in the belt and the strength of the belt. The coefficient of friction is 0.25 and the angle of lap is 220°.*

Solution $P = (T_1 - T_2)v$

or $9600 = (T_1 - T_2) \times 6$

or $T_1 - T_2 = 1600$ (i)

$$\frac{T_1}{T_2} = e^{\mu\theta} = e^{0.25 \times \frac{220\pi}{180}} = e^{0.96} = 2.61$$

or $T_1 = 2.61\,T_2$ (ii)

From (i) and (ii),

$2.61\,T_2 - T_2 = 1600$

$T_2 = 994$ N

$T_1 = 2594$ N

Centrifugal tension $= mv^2 = 1 \times 12^2 = 144$ N

Initial tension $T_o = \frac{T_1 + T_2}{2} + T_c$

$= \frac{2594 + 994}{2} + 144 = 1938$ N

Strength of the belt = Total tension on the tight side

$= T_1 + T_c$

$= 2594 + 144$

$= 2738$ N

Example 9.16 *In an open-belt drive, the diameters of the larger and the smaller pulleys are 1.2 m and 0.8 m respectively. The smaller pulley rotates at 320 rpm. The centre distance between the shafts is 4 m. When stationary, the initial tension in the belt is 2.8 kN. The mass of the belt is 1.8 kg/m and the coefficient of friction between the belt and the pulley is 0.25. Determine the power transmitted.*

Solution Let smaller pulley be the driving pulley.

$N_1 = 320$ rpm

$D = 1.2$ m; $R = 0.6$ m; $d = 0.8$ m; $r = 0.4$ m;

$C = 4$ m; $\mu = 0.25$; $m = 1.8$ kg/m; $T_o = 2800$ N

$$v = \frac{\pi d N_1}{60} = \frac{\pi \times 0.8 \times 320}{60} = 13.4 \text{ m/s}$$

Also $T_c = mv^2 = 1.8 \times 13.4^2 = 323.4$ N

$$\text{Initial tension } T_o = \frac{T_1 + T_2}{2} + T_c$$

$$\text{or } 2800 = \frac{T_1 + T_2}{2} + 323.4 \text{ or } T_1 + T_2 = 4953 \text{ N} \quad \text{(i)}$$

$$\text{Now, } \theta = \pi - 2\sin^{-1}\left(\frac{R - r}{C}\right)$$

$$= \pi - 2\sin^{-}\left(\frac{0.6 - 0.4}{4}\right) = \pi - 5.73° = \pi - 0.01$$

$$= 3.042 \text{ rad}$$

$$\text{Also} \quad \frac{T_1}{T_2} = e^{\mu\theta} \text{where } \frac{T_1}{T_2} = e^{0.25 \times 3.042} = 2.14$$

or $\quad T_1/T_2 = 2.14\ T_2$ (ii)

From (i) and (ii), $T_2 = 1577$ N and $T_1 = 3376$ N

$P = (T_1 - T_2)v = (3376 - 1577) \times 13.4 = 24106$ W or 24.106 kW

Example 9.17 *The initial tension in a belt drive is found to be 600 N and the ratio of friction tensions is 1.8. The mass of the belt is 0.8 kg/m length. Determine the*

(i) velocity of the belt for maximum power transmission

(ii) tension on the tight side of the belt when it is started

(iii) tension on the tight side of the belt when running at maximum speed

Solution From equation for maximum power transmission with consideration of initial tension,

$$\text{(i)} \quad v = \sqrt{\frac{T_o}{3m}} = \sqrt{\frac{600}{3 \times 0.8}} = 15.8 \text{ m/s}$$

(ii) Tension on the tight side of the belt when it is started

$$T_1 = \frac{2kT_o}{k+1} = \frac{2 \times 1.8 \times 600}{1.8 + 1} = 771.4 \text{ N}$$

(iii) Tension on the tight side of the belt when running at maximum speed.

Centrifugal tension $= mv^2 = 0.8 \times 15.8^2$

$= 199.7$ N

$$\text{Initial tension } T_o = \frac{T_1 + T_2}{2} + T_c$$

$$600 = \frac{T_1 + (T_1/1.8)}{2} + 199.7$$

$0.778\ T_1 = 400.3$

$T_1 = 514.6$ N

It can also be found by applying the relation

$$T_1 = \frac{2k(T_o - T_c)}{k+1} = \frac{2 \times 1.8(600 - 199.7)}{1.8 + 1}$$

$= 514.6$ N

9.17 CREEP

It is seen that when a belt moves over the driving pulley, tension in the belt decreases from T_1 to T_2.

Let l_1 = stretch in unit length of belt due to T_1

Let l_2 = stretch in unit length of belt due to T_2

Assuming that the strain in the belt is proportional to the stress in it,

As $\quad T_1 > T_2$

Therefore, $\quad l_1 > l_2$

Thus, a length $(1 + l_1)$ of belt approaches the driving pulley and a length $(1 + l_2)$ leaves it. As $(1 + l_1)$ is greater than $(1 + l_2)$, the belt slips back over the driving pulley. This slip is known as the *creep* of the belt.

On the driven pulley, the belt tension increases from T_2 to T_1. This means a shorter length $(1 + l_2)$ approaches the driving pulley and a greater length $(1 + l_1)$ leaves it. Thus, the belt creeps forward by an amount $(l_1 - l_2)$. This makes the driven pulley to move at a slower speed than the belt.

Thus, the effect of creep is to slow down the speed of the belt on the driving pulley than that of the rim of the pulley and to reduce the rim velocity of the driven pulley than that of the belt on it. Therefore, the net effect of creep is to reduce the speed of the driven pulley than what it would have been without creep and Thus, reducing the power transmitted.

Let σ_1 = stress on the tight side of the belt

σ_2 = stress on the slack side of the belt

ε_1 = strain on the tight side

ε_2 = strain on the slack side

N_1 = speed of the driving pulley

N_2 = speed of the driven pulley

l = original length of the belt

E = modulus of elasticity of the belt material

Assuming that the stress strain curve for the belt to be parabolic in nature,

$$\varepsilon_1 = \frac{\sqrt{\sigma_1}}{E} \quad \text{and} \quad \varepsilon_2 = \frac{\sqrt{\sigma_2}}{E}$$

Length on the tight side = $l + l.\varepsilon_1$

Length on the slack side = $l + l.\varepsilon_2$

$$\text{Velocity ratio} = \frac{V_2}{V_1} = \frac{\text{peripheral speed of driven pulley}}{\text{peripheral speed of driving pulley}}$$

The peripheral speed of a pulley is to be proportional to the approaching length of belt. Thus,

$$\frac{\pi D_2 N_2}{\pi D_1 N_1} = \frac{l + l\varepsilon_2}{l + l\varepsilon_1}$$

$$\frac{N_2}{N_1} = \frac{D_1}{D_2} \cdot \frac{1 + \varepsilon_2}{1 + \varepsilon_1} = \frac{D_1}{D_2} \cdot \left[\frac{E + \sqrt{\sigma_2}}{E + \sqrt{\sigma_1}} \right]$$

Example 9.18 *The driving pulley of an open belt drive is of 800 mm diameter and rotates at 320 rpm while transmitting power to a driven pulley of 250 mm diameter. The Young's modulus of elasticity of the belt material is 110 N/mm^2. Determine the speed lost by the driven pulley due to creep if the stresses in the tight and slack sides of the belt are found to be 0.8 N/mm^2 and 0.32 N/mm^2 respectively.*

Solution Larger pulley is the driving pulley, N_1 = 320 rpm

$\therefore D = 800$ rpm; $d = 250$ rpm

If creep is neglected, $\frac{N_2}{N_1} = \frac{D}{d}$

or $N_2 = N_1 \times \frac{D}{d} = 320 \times \frac{800}{250} = 1024$ rpm

$$\frac{N_2}{N_1} = \frac{D}{d} \cdot \left[\frac{E + \sqrt{\sigma_2}}{E + \sqrt{\sigma_1}}\right]$$

$$N_2 = 320 \times \frac{800}{250} \cdot \left[\frac{110 + \sqrt{0.32}}{110 + \sqrt{0.8}}\right]$$

$= 320 \times 3.2 \times 0.997$

$= 1021$ rpm

Speed lost = 1024 – 1021 = 3 rpm

9.18 CHAINS

A chain is regarded in between the gear drive and the belt drive. Like gears, chains are made of metal and, therefore, occupy lesser space and give constant velocity ratios. Like belts, they are used for longer centre distances.

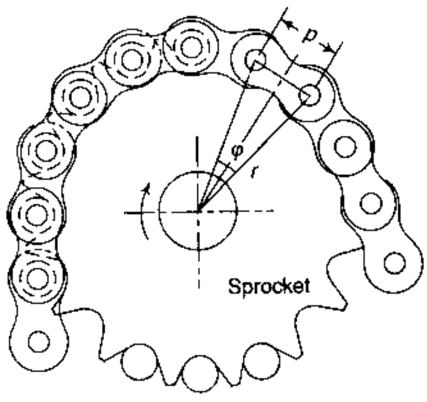


Fig. 9.17

A chain drive of a machine

Advantages

- Constant velocity ratio due to no slip and thus, it is a positive drive.
- No effect of overloads on the velocity ratio.
- Oil or grease on surfaces does not affect the velocity ratio.
- Chains occupy less space as these are made of metals.
- Lesser loads are put on the shafts.
- High transmission efficiency due to no slip.
- Through one chain only, motion can be transmitted to several shafts.

Disadvantages

- It is heavier as compared to the belt.
- There is a gradual stretching and increase in length of chains. From time to time some of its links have to be removed.
- Lubrication of its parts is required.
- Chains are costlier as compared to belts.

The wheels over which chains are run, corresponding to the pulleys of a belt drive, are known as *sprockets*. The surfaces of sprockets conform to the type of chain used. Usually, a sprocket has projected teeth that fit into the recesses in the chain. Thus, the chain passes round the sprockets as a series of chordal links (Fig. 9.17).

The distance between roller centres of two adjacent links is known as the pitch (p) of the chain. A circle through the roller centres of a wrapped chain round a sprocket is called the *pitch circle* and its diameter as *pitch circle diameter*.

Observe that a chain is wrapped round the sprocket in the form of a pitch polygon and not in the form of a pitch circle.

Let T = number of teeth on a sprocket.

φ = angle subtended by chord of a link at the centre of sprocket

r = radius of the pitch circle

Then

$$p = 2r\sin\frac{\varphi}{2} = 2r\sin\frac{1}{2}\left(\frac{360^\circ}{T}\right) = 2r\sin\frac{180^\circ}{T}$$

or

$$r = \frac{p}{2\sin\dfrac{180^\circ}{T}} = \frac{p}{2}\operatorname{cosec}\frac{180^\circ}{T} \tag{9.21}$$

9.19 CHAIN LENGTH

For a given pair of sprockets at a fixed distance apart, the length of the chain may be calculated in the same way as for an open belt. Since the pitch line of a sprocket is a polygon, Eq. 9.6 will give a length slightly more than the actual length.

Let R and r be the radii of the pitch circles of the two sprockets having T and t teeth respectively. Also,

let L = length of the chain

C = centre distance between sprockets = kp

p = pitch of chain

From Eq. (9.6),

$$L = \pi(R + r) + \frac{(R - r)^2}{C} + 2C$$

The first term in the equation is half the sum of the circumference of the pitch circles. In case of a chain it will be $(pT + pt)/2$.

Replacing R and r in the second term by

$$R = \frac{p}{2}\operatorname{cosec}\frac{180^\circ}{T} \text{ and } r = \frac{p}{2}\operatorname{cosec}\frac{180^\circ}{t}$$

$$L = \frac{pT + pt}{2} + \frac{\left(\dfrac{p}{2}\operatorname{cosec}\dfrac{180^\circ}{T} - \dfrac{p}{2}\operatorname{cosec}\dfrac{180^\circ}{t}\right)^2}{kp} + 2kp$$

$$= p\left[\frac{T + t}{2} + \frac{\left(\operatorname{cosec}\dfrac{180^\circ}{T} - \operatorname{cosec}\dfrac{180^\circ}{t}\right)^2}{4k} + 2k\right] \tag{9.22}$$

Note that the terms in the square bracket must be an integral number of pitch lengths. In case it is a fraction, it must be rounded off to the next integral number.

9.20 ANGULAR SPEED RATIO

The chain is wrapped round the sprocket in the form of a pitch polygon and not as a pitch circle. From Fig. 9.18, it may be observed that the axial line of the chain vibrates between two positions shown by full and dotted lines.

Even if the sprocket rotates at an uniform angular velocity ω, the linear velocity of the chain will be varying from a maximum $\omega.AC$ to a minimum $\omega.AD$. Thus, the magnitude of the speed variation is the ratio of the distances AC to AD. The variation in the chain speed also causes a variation in the angular speed of the driven sprocket. However, by increasing the number of teeth on the sprocket, the magnitude of the variation in speed may be minimized.

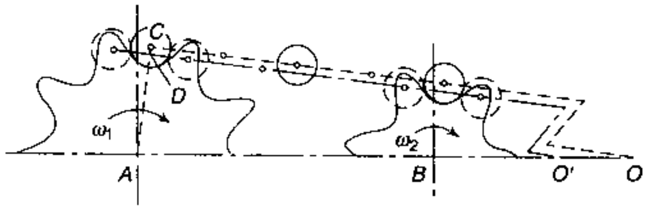


Fig. 9.18

It can be shown that at any instant, if the line of transmission cuts the line of centres at O, the angular velocities of the two sprockets will be in the inverse ratio of the distances of their centres from O, i.e.,

$$\frac{\omega_2}{\omega_{11}} = \frac{OA}{OB}$$

The variation of ω_2 will be between

$$\omega_1 \frac{OA}{OB} \text{ and } \omega_1 \frac{O'A}{O'B}$$

Thus, at any instant, the angular velocity of the driven shaft would be changing.

Example 9.19 *The center-to-centre distance between the two sprockets of a chain drive is 600 mm. The chain drive is used to reduce the speed from 180 rpm to 90 rpm on the driving sprocket has 18 teeth and a pitch circle diameter of 480 mm. Determine the*

(i) number of teeth on the driven sprocket

(ii) pitch and the length of the chain

Solution (i) $\frac{N_2}{N_1} = \frac{T_1}{T_2}$

or $T_2 = T_1 \frac{N_1}{N_2} = 18 \times \frac{180}{90} = \underline{36}$

(ii) $p = 2r \sin \frac{180°}{T} = 2 \times 0.24 \times \sin \frac{180°}{36}$

$= 0.0418$ m or $\underline{41.8 \text{ mm}}$

$k = C/p = 0.600/0.0418 = 14.342$

$$L = p\left[\frac{T+t}{2} + \frac{\left(\operatorname{cosec}\frac{180°}{T} - \operatorname{cosec}\frac{180°}{t}\right)^2}{4k} + 2k\right]$$

$$= 0.0418\left[\frac{36+18}{2} + \frac{\left(\operatorname{cosec}\frac{180°}{36} - \operatorname{cosec}\frac{180°}{18}\right)^2}{4 \times 14.342} + 2 \times 14.342\right]$$

$= 0.0418 \times (27 + 0.569 + 28.684)$

$= \underline{2.351 \text{ m}}$

9.21 CLASSIFICATION OF CHAINS

Chains have been classified into hoisting chains, conveyor chains and power-transmission chains. Each type has been discussed below:

1. Hoisting Chains

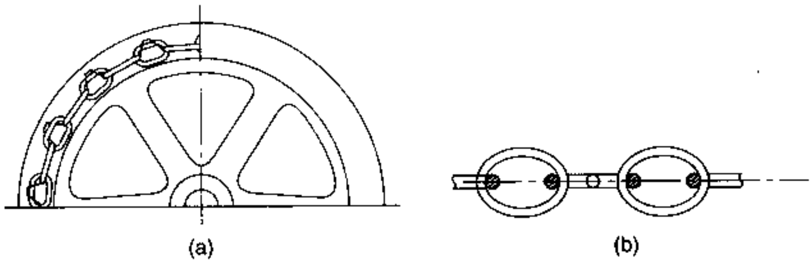


Fig. 9.19

Hoisting chains include *oval-link* and *stud-link* chains. An oval-link chain is a common form of hoisting chain [Fig. 9.19(a)]. It consists of oval links and is also known as *coil chain*. Such chains are used for lower speeds only.

Figure 9.19(b) shows a stud-link chain. This does not link or tangle easily.

2. Conveyor Chains

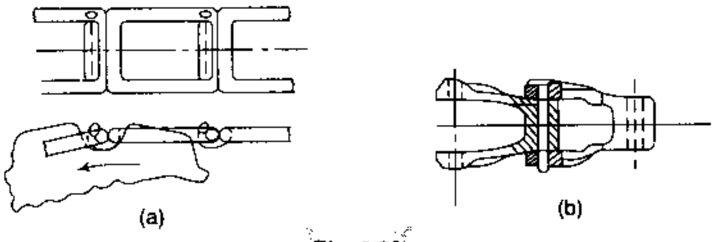


Fig. 9.20

Conveyor chains may be of detachable or *hook-joint* type [Fig. 9.20(a)], or of the *closed-end pintle* type [Fig. 9.20(b)]. The sprocket teeth are so shaped and spaced that the chain should run onto and off the sprocket smoothly and without interference. Such chains are used for low-speed agricultural machinery. The material of the links is, usually, malleable cast iron. The motion of the chain is not very smooth.

3. Power Transmissions Chains

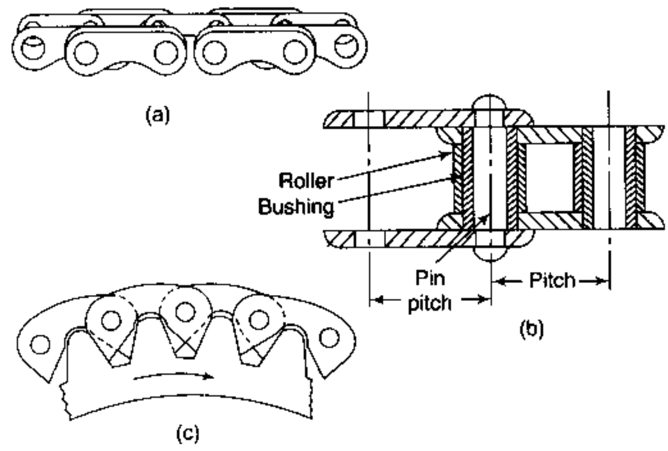


Fig. 9.21

These chains are made of steel in which the wearing parts are hardened. They are accurately machined and run on carefully designed sprockets.

These are of three types:

(i) *Block Chain* This type of chain [Fig. 9.21(a)] is mainly used for transmission of power at low speeds. Sometimes, they are also used as conveyor chains in place of malleable conveyor chains.

(ii) *Roller Chain* A common form of a roller chain is shown in [Fig. 9.21(b)]. A bushing is fixed to the inner link whereas the outer link has a pin fixed to it. There is only sliding motion between the pin and the bushing. The roller is made of a hardened material and is free to turn on the bushing. Figure 9.17 shows this type of chain in place on the sprocket. A good roller chain is quieter and wears less as compared to a block chain.

(iii) *Silent Chain (Inverted Tooth Chain)* Though roller chains can run quietly at fairly high speeds, the silent chains or inverted tooth chains are used where maximum quietness is desired.

Silent chains do not have rollers. The links are so shaped as to engage directly with the sprocket teeth. The included angle is either 60° or 75° [Fig. 9.21(c)].

Summary

1. Power is transmitted from one shaft to another by means of belts, ropes, chains and gears.
2. Belts, ropes and chains are used where the distance between the shafts is large. For small distances, gears are preferred.
3. Belts and ropes transmit power due to friction between them and the pulleys. If the power transmitted exceeds the force of friction, the belt or rope slips over the pulley.
4. Belts and ropes are strained during motion as tensions are developed in them.
5. Owing to slipping and straining action, belts and ropes are not positive type of drives, i.e., their velocity ratios are not constant.
6. The effect of slip is to decrease the speed of the belt on the driving shaft and to decrease the speed of the driven shaft.
7. A belt may be of rectangular section, known as a *flat belt* or of trapezoidal section, known as a *V-belt*.
8. In case of a flat belt, the rim of the pulley is slightly *crowned* which helps to keep the belt running centrally on the pulley rim.
9. The groove on the rim of the pulley of a V-belt drive is made deeper to take the advantage of the wedge action. The belt does not touch the bottom of the groove.
10. A multiple V-belt system, using more than one belt in the two pulleys, can be used to increase the power transmitting capacity.
11. An open-belt drive is used when the driven pulley is to be rotated in the same direction as the driving pulley and a crossed-belt drive when in the opposite direction.
12. While transmitting power, one side of the belt is more tightened (known as tight side) as compared to the other (known as slack side).
13. Velocity ratio is the ratio of speed of the driven pulley to that of the driving pulley.
14. Usual materials of flat belts are leather, canvas, cotton and rubber.
15. V-belts are made of rubber impregnated fabric with angle of V between 30 to 40 degrees.
16. The materials for ropes are cotton, hemp, manila or wire.
17. The main types of pulleys are *idler, intermediate* (or *countershaft), loose and fast and guide*.
18. *Law of belting* states that the centre line of the belt when it approaches a pulley must lie in the mid-plane of that pulley. However, a belt leaving a pulley may be drawn out of the plane of the pulley.
19. The length of belt depends only on the sum of the pulley radii and the centre distance in case of crossed-belt drive whereas it depends on the sum as well as the difference of the pulley radii apart from the centre distance in case of open-belt drive.
20. A cone pulley has different sets of pulley radii to give varying speeds of the driven shaft.
21. The ratio of belt tensions when the belt is on the point of slipping on the pulleys, $\frac{T_1}{T_2} = e^{\mu\theta}$ for flat belt drive,

    $\frac{T_1}{T_2} = e^{\mu\theta/\sin\alpha}$ for V-belt drive
22. Power transmitted is, $P = (T_1 - T_2) \times v$
23. The centrifugal force produces equal tensions on the two sides of the belt, i.e., on the tight side as well as on the slack side. It is independent of the tight and slack side tensions and depends only on the velocity of the belt over the pulley.
24. For maximum power transmission, centrifugal tension in the belt must be equal to one-third of the maximum allowable belt tension and the belt should be on the point of slipping.
25. Initial tension in the belt is given by, $T_0 = \frac{T_1 + T_2}{2}$
26. As more length of belt approaches the driving pulley than the length that leaves, the belt slips back over the driving pulley. This slip is known as *creep* of the belt.

Exercises

1. What are different modes of transmitting power from one shaft to another? Compare them.
2. Discuss the effect of slip of belt on the pulleys on the velocity ratio of a belt drive.
3. Name the materials of the flat belts, V-belts and ropes.
4. What do you mean by crowning of pulleys in flat-belt drives? What is its use?
5. What are different types of pulleys? Explain briefly.
6. Explain the following:
   (i) Idler pulleys
   (ii) Intermediate pulleys
   (iii) Loose and fast pulleys
   (iv) Guide pulleys
7. Define and elaborate the law of belting.
8. Deduce expressions for the exact and approximate lengths of belt in an open-belt drive.
9. What is meant by cross-belt drive? Find the length of belt in a cross-belt drive.
10. Where do we use cone (stepped) pulleys? Explain the procedure to design them.
11. Derive the relation for ratio of belt tensions in a flat-belt drive.
12. Derive the relation, $\frac{T_1}{T_2} = e^{\mu\theta}$ for a flat-belt drive with usual notations.
13. Deduce an expression for the ratio of tight and slack side tensions in case of a V-belt drive.
14. What is the effect of centrifugal tension on the tight and slack sides of a belt drive? Show that it is independent of the tight and slack-side tensions and depends only on the velocity of the belt over the pulley.
15. What is the effect of centrifugal tension on the power transmitted?
16. Derive the condition for maximum power transmission by a belt drive considering the effect of centrifugal tension.
17. What is meant by initial tension in a belt drive?
18. What is creep in a belt drive?
19. A motor shaft drives a main shaft of a workshop by means of a flat belt, the diameters of the pulleys being 500-mm and 800-mm respectively. Another pulley of 600 mm diameter on the main shaft drives a counter-shaft having a 750-mm diameter pulley. If the speed of the motor is 1600 rpm, find the speed of the countershaft neglecting the thickness of the belt and considering a slip of 4% on each drive.
   (737.3 rpm)
20. Two pulleys on two shafts are connected by a flat belt. The driving pulley is 250 mm in diameter and runs at 150 rpm. The speed of the driven pulley is to be 90 rpm. The belt is 120 mm wide, 5-mm thick and weighs 1000 kg/m^3. Assuming a slip of 2% between the belt and each pulley, determine the diameter of the driven pulley. Also, find the total effective slip.
   (403 mm; 3.96%)
21. The pulleys of two parallel shafts that 8 m apart are 600 mm and 800 mm in diameters and are connected by a crossed belt. It is needed to change the direction of rotation of the driven shaft by adopting the open-belt drive. Calculate the change in length of the belt.
   (Shorten the belt by 60 mm)
22. Determine the diameters of the cone pulley joined by a crossed belt. The driven shaft is desired to be run at speeds of 60, 90 and 120 rpm while the driving shaft rotates at 160 rpm. The centre distance between the axes of the two shafts is 2.5 m. The smallest pulley diameter can be taken as 150 mm.
   (150 mm and 400 mm; 198 mm and 352 mm; 236 mm and 314 mm)
23. Design a set of stepped pulleys to drive a machine from a countershaft running at 300 rpm. It is needed to have the following speeds of the driven shaft: 140 rpm, 180 rpm and 220 rpm. The centre distance between the axes of the two shafts is 5 m. The diameter of the smallest pulley is 300 mm. The two shafts rotate in the same direction.
   (300 mm and 642 mm; 354 mm and 590 mm; 400 mm and 545 mm)
24. A countershaft is to be driven at 240 rpm from a driving shaft rotating at 100 rpm by an open-belt drive. The diameter of the driving pulley is 480 mm. The distance between the centre line of shafts is 2 m. Find the width of the belt to transmit 3 kW of power if the safe permissible stress in tension is 15 N/mm width of the belt. Take $\mu = 0.3$.
   (134 mm)
25. A casting having a mass of 100 kg is suspended freely from a rope. The rope makes 2 turns round a drum of 300 mm diameter rotating at 24 rpm. The other end of the rope is pulled by a man. Calculate

the force required by the man, power to raise the casting and the power supplied by drum run by a prime-mover. Take μ = 0.3.

(226 N; 3.698 kW; 3.613 kW)

26. A leather belt transmits 10 kW from a motor running at 600 rpm by an open-belt drive. The diameter of the driving pulley of the motor is 350 mm, centre distance between the pulleys is 4 m and speed of the driven pulley is 180 rpm. The belt weighs 1100 kg/m^3 and the maximum allowable tension in the belt is 2.5 N/mm^2. μ = 0.25. Find the width of the belt assuming the thickness to be 10 mm. Neglect the belt thickness to calculate the velocities.

(73.8 mm)

27. Two pulleys mounted on two parallel shafts that are 2 m apart are connected by a crossed belt drive. The diameters of the two pulleys are 500 mm and 240 mm. Find the length of the belt and the angle of contact between the belt and each pulley. Also, find the power transmitted if the larger pulley rotates at 180 rpm and the maximum permissible tension in the belt is 900 N. The coefficient of friction between the belt and pulley is 0.28.

(5.23 m, 201.4°, 2.658 kW)

28. Determine the maximum power that can be transmitted through a flat belt having the following data:

X-section of the belt = 300 mm × 12 mm
Ratio of friction tensions = 2.2
Maximum permissible tension in belt = 2 N/mm^2
Mass density of the belt material = 0.0011 g/mm^3

(64.46 kW)

29. A V-belt weighting 1.6 kg/m run has an area of cross-section of 750 mm^2. The angle of lap is 165° on the smaller pulley which has a groove angle of 40°. μ = 0.12. The maximum safe stress in the belt is 9.5 N/mm^2. What is the power that can be transmitted by the belt at a speed of 20 m/s?

(82.485 kW)

30. A leather belt transmits 8 kW of power from a pulley that is 1.1 m in diameter running at 200 rpm. The angle of lap is 160° and the coefficient of friction between belt and pulley is 0.25. The maximum safe working stress in the belt is 2.2 N/mm^2. The thickness of the belt is 8 mm and the density of leather is 0.001 g/mm^3. Find the width of the belt taking centrifugal tension into account.

(78.6 mm)

31. A rope drive transmits 40 kW at 120 rpm by using 15 ropes. The angle of lap on the smaller pulley which is 300 mm in diameter is 165°. Coefficient of friction is 0.25 and the angle of groove is 40°. The rope weighs (50×10^{-6}) G^2 kg per metre length of rope and the working tension is limited to 0.14 G^2 N where G is the girth (circumference) of rope in mm. Determine the initial tension and the diameter of each rope.

(903.8 N; 34.2 mm)

32. The smaller pulley of a flat belt drive has a radius of 220 mm and rotates at 480 rpm. The angle of lap is 155°. The initial tension in the belt is 1.8 kN and the coefficient of friction between the belt and the pulley is 0.3. Determine the power transmitted by the belt.

(15.3 kW)

33. A rope drive uses ropes weighing 1.6 kg/m length. The diameter of the pulley is 3.2 m and has 12 grooves of 40° angle. The coefficient of friction between the ropes and the groove sides is 0.3 and the angle of contact is 165°. The permissible tension in the ropes is 870 N. Determine the speed of the pulley and the power transmitted.

(80.3 rpm, 86.18 kW)

34. A man wants to lower an engine weighting 380 kg from a trolley to the ground by using a rope which he passes over a fixed horizontal pipe overhead. The man is capable of controlling the motion with a force of 200 N or less on the free end of the rope. Find the minimum number of times the rope must be passed round the pipe if μ = 0.22.

(2.1 turns)

35. A chain drive is used for speed reduction from 240 rpm to 110 rpm. The number of teeth on the driving sprocket is 22. The centre to centre distance between two sprockets is 540 mm and the pitch circle diameter of the driven sprocket is 480 mm. Determine the number of teeth on the driven sprocket, pitch and the length of the chain.

(48, 31.4 mm, 2.21 m)

10 GEARS

Introduction

Gears are used to transmit motion from one shaft to another or between a shaft and a slide. This is accomplished by successively engaging teeth.

Gears use no intermediate link or connector and transmit the motion by direct contact. In this method, the surfaces of two bodies make a tangential contact. The two bodies have either a rolling or a sliding motion along the tangent at the point of contact. No motion is possible along the common normal as that will either break the contact or one body will tend to penetrate into the other.

If power transmitted between two shafts is small, motion between them may be obtained by using two plain cylinders or discs 1 and 2 as shown in Fig. 10.1. If there is no slip of one surface relative to the other, a definite motion of 1 can be transmitted to 2 and vice-versa. Such wheels are termed as *friction wheels*. However, as the power transmitted increases, slip occurs between the discs and the motion no longer remains definite.

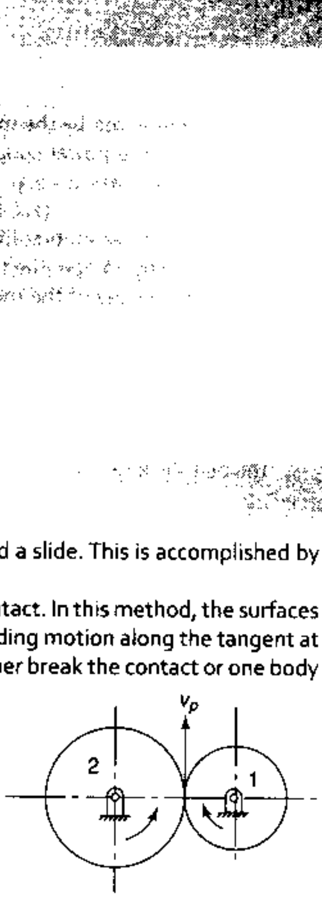


Fig. 10.1

Assuming no slipping of the two surfaces, the following kinematic relationship exists for their linear velocity:

$$v_p = \omega_1 r_1 = \omega_2 r_2$$
$$= 2\pi N_1 r_1 = 2\pi N_2 r_2$$

or
$$\frac{\omega_1}{\omega_2} = \frac{N_1}{N_2} = \frac{r_2}{r_1} \qquad (10.1)$$

where N = angular velocity (rpm)
ω = angular velocity (rad/s)
r = radius of the disc

Subscripts 1 and 2 represent discs 1 and 2 respectively.

The relationship shows that the speeds of the two discs rolling together without slipping are inversely proportional to the radii of the discs.

To transmit a definite motion of one disc to the other or to prevent slip between the surfaces, projections and recesses on the two discs can be made which can mesh with each other. This leads to the formation of teeth on the discs and the motion between the surfaces changes from rolling to sliding. The discs with teeth are known as *gears* or *gear wheels*.

It is to be noted that if the disc 1 rotates in the clockwise direction, 2 rotates in the counter-clockwise direction and vice-versa.

Although large velocity ratios of the driving and the driven members have been obtained by the use of gears, practically, it is limited to 6 for spur gears and 10 for helical and herringbone gears. To obtain large reductions, two or more pairs of gears are used.